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New BPS Vortices in Maxwell-Chern-Simons-Higgs Model with Neutral Scalar Field

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Abstract. Vortices are topological defects that exist in the planar dimension. In this paper, we present first-order formalism to Maxwell-Chern-Simons-Higgs model with general coupling functions using the BPS Lagrangian method. We obtain finite an energy solution with the potential that depends on the generalized functions $w(|\phi|, N)$ and $h(|\phi|, N)$. We introduce some particular generalized functions and present the numerical solution to the obtained BPS equations. We find that the vortex solution of our model does not have electric field and the energy density forms a ring-like structure. Using the conserved local $U(1)$ current, we obtain that the charge density of this model is localized and the charge itself is proportional to the magnetic flux.

Keywords: *BPS equations; BPS Lagrangian; Maxwell-Chern-Simons-Higgs; topological defects; vortex.*

1 Introduction

Topological defects have become an interesting topic of discussion in recent years. This type of solutions which arise from a nonlinear model can be interpreted as a type of particles that is different from the usual elementary particles in the Standard Model. They have distinct topological property from their vacuum such that there are no physical processes with finite amount of energy which can deform these solutions into their vacuum. One particular kind of topological defects in planar space is known as vortices, Manton *et al.* and Weinberg [1, 2]. There exist two kind of vortices, namely global and gauged vortices, each corresponds to the global and local $U(1)$ transformation respectively. The generalization of global vortices in 3+1 dimensional spacetime have their application in cosmology. Kibble mechanism explains how the formation of cosmic strings may occurs in the early universe during the course of symmetry-breaking cosmological phase transitions, Weinberg and Kibble [2, 3]. Gauged vortices also have their own interesting application in condensed matter physics, mainly in the superconductor of the second group that has been studied by Abrikosov in [4].

The standard kinetic term of the gauge field in gauged vortices is the Maxwell Lagrangian. However, in the odd-dimensional spacetime, there exist another possible kinetic term for the gauge field that is Lorentz invariant, local, and remain gauge invariant up to its boundary term, that is the Chern-Simons term, Dunne [5]. In planar dimension, coupling between Chern-Simons term and matter field give rise into a new kind of particles which have distinct statistics from the ordinary bosons and fermions. It was studied by Wilczek in [6] and is called anyons.

Mathematically, it is possible to consider a model in which the kinetic terms of the gauge field are described both by the Maxwell and Chern-Simons term. In one of the earliest studies, Lee *et al.* [7] shows that coupling between Maxwell-Chern-Simons term and the Higgs field (MCSH) give rise into electrically charged vortices. The first differential formulation in this study leads into a conclusion that the self-dual solution is static and there is an identification between the scalar gauge field and the neutral scalar field, removing the gauge invariant problem. This study is generalized by Bazeia *et al.* in [7] by adding generalized coupling functions. In this generalization, coupling functions between Maxwell and the kinetic term of neutral scalar field remain identical, which makes the scalar gauge and neutral scalar field remain identical. Different approach was done by Torres in Ref. [8] by introducing anomalous magnetic moment to the coupling between gauge and Higgs field. This addition makes the second order dynamical equation for the gauge field to be satisfied by first order differential equation. This enables the temporal gauge field to be written in terms of the Higgs field, thus removing the gauge invariant problem. Generalization of this model was done in a quite similar manner as in the previous model[1] was done by Andreade *et al.* [9]. Recent study by Andreade *et al.* [10] shows that vortex solution may exist in the generalized MCSH model even with no neutral scalar field and minimal coupling between gauge and the Higgs field. Approach to this model was done by considering the stressless condition. In this study, first-order differential equation is introduced in the analysis such that is satisfy stressless condition and the equation of motion. The consequence of this is that the equation which relates scalar gauge field to the Higgs field is obtained.

We learn that in the previous studies, the identification between temporal gauge field and a scalar field is need to be done to avoid violating gauge invariant

[1] The one with neutral scalar field.

condition. In this paper, we attempt to obtain vortex solution with all the involved fields independent to one another. The method that we will use to obtain the first-order formalism is the BPS Lagrangian method introduced by Ardian in [11]. This paper consists of four sections. In Sec. II, we introduce the model and apply the radially symmetric ansatz into it. In Sec. III, we implement the BPS Lagrangian method into the model and analyze the obtained equations numerically. Finally, we conclude this study in Sec. IV where we give our final comment and discuss the possibility for further research.

2 Generalized Maxwell-Chern-Simons-Higgs Model

In this paper, we consider the most general version of the model with the Lagrangian density

$$
\begin{aligned}
L = &-\frac{h(|\phi|,N)}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\kappa}{4}\epsilon^{\mu\nu\rho}A_{\mu}F_{\nu\rho} + w(|\phi|,N)\,|D_{\mu}\phi|^{2} \\
&+\frac{G(|\phi|,N)}{2}\partial_{\mu}N\partial^{\mu}N - V(|\phi|,N)\ ,
\end{aligned}
\tag{1}
$$

where $h(|\phi|,N)$, $w(|\phi|,N)$, and $G(|\phi|,N)$ are the generalized coupling functions that satisfy positive-definite and dimensionless condition, $V(|\phi|,N)$ is a general potential with non-negative value, $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$ is the abelian gauge curvature tensor, and $D_{\mu}\phi = \partial_{\mu}\phi + ieA_{\mu}\phi$ describes the minimal coupling between Higgs and the gauge field.

The equation of motions for the fields in this model are given by the Euler-Lagrange equations. For the gauge field, we have

$$
\partial_{\mu}(hF^{\mu\nu}) + J^{\nu} = \kappa F^{\nu}\ , \tag{2}
$$

where

$$
J^{\nu} = iew\left(\phi\overline{D^{\nu}\phi} - \bar{\phi}D^{\nu}\phi\right), \tag{3}
$$

is the $U(1)$ conserved current and

$$
F^{\nu} = (1/2)\epsilon^{\nu\rho\sigma}F_{\rho\sigma}, \tag{4}
$$

is the dual of gauge curvature tensor. From eq. (2), one can observe that the temporal gauge, $A_0 = 0$, cannot be used since it will lead to the trivial solution. The remaining equation of motions to Lagrangian (1) can be written as

$$D_\mu\left(wD^\mu\phi\right)+\frac{1}{2}\partial_{\bar{\phi}}h\left(F_\mu F^\mu-\partial_\mu N\partial^\mu N\right)-\partial_{\bar{\phi}}w\,|\,D_\mu\phi\,|^2$$
$$+\partial_{\bar{\phi}}V=0\ , \tag{5}$$

$$\partial_\mu\left(G\partial^\mu N\right)+\frac{1}{2}\left(\partial_N h\ F_\mu F^\mu-\partial_N G\ \partial_\mu N\partial^\mu N+2\partial_N w\,|\,D_\mu\phi\,|^2\right)$$
$$+\partial_N V=0\ , \tag{6}$$

We may also calculate the energy-momentum tensor for the latter calculational purpose.

$$T^{\mu\nu}=h(|\,\phi\,|,N)\left(F^{\mu\rho}F_\rho^\nu+\frac{1}{4}\eta^{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}\right)+\eta^{\mu\nu}V(|\,\phi\,|,N)$$
$$+G(|\,\phi\,|,N)\left(\partial^\mu N\partial^\nu N-\frac{1}{2}\eta^{\mu\nu}\partial_\rho N\partial^\rho N\right)\ , \tag{7}$$

We are interested to the radially-symmetric static solution of the form

$$\phi(r,\theta)=vg(r)e^{in\theta};\quad N\equiv N(r);\quad A_0\equiv A_0(r), \tag{8a}$$

$$\vec{A}(r,\theta)=-\frac{\hat{\theta}}{er}\left(a(r)-n\right), \tag{8b}$$

with v being the vacuum expectation value of Higgs field and $n=\pm1,\pm2,\dots$ is the winding number. Substituting ansatz (8) into the Lagrangian density (1), we obtain

$$\mathrm{L}_{eff}=\frac{h}{2}A'_0(r)^2-\frac{h}{2}\left(\frac{a'(r)}{er}\right)^2-v^2w\left(g'(r)^2+\frac{a^2g^2}{r^2}\right)$$
$$-\frac{G}{2}N'(r)^2+\frac{\kappa}{2er}\left(A'_0(r)(a-n)-A_0(r)\right) \tag{9}$$
$$+e^2v^2A_0^2g^2w-V.$$

We may also take the 00-component of the energy-momentum tensor and write it in terms of the defined ansatz so that the energy density of this model is

$$\rho = \frac{h}{2}\left(A'_0(r)^2 + \frac{a'^2}{e^2 r^2} \right) + v^2 w \left(g'(r)^2 + \frac{a^2 g^2}{r^2} + e^2 A_0^2 g^2 \right) \\ + \frac{G}{2} N'(r)^2 + V, \tag{10}$$

where we have $w \equiv w(g,N)$, $h \equiv h(g,N)$, $V \equiv V(g,N)$, and the primed function denotes its derivative with respect to r.

3 BPS Equations Formalism

3.1 BPS Lagrangian Method

It has been shown in [11] that the BPS Lagrangian method is sufficient to derive the BPS equations for vortices in the standard and generalized Maxwell-Higgs and Born-Infeld-Higgs models. Furthermore, the method has been used for the case of Skyrme model [12]. In the BPS Lagrangian method, we basically trying to rewrite the original Lagrangian density into quadratic terms that consist of all first-derivative of the fields by subtracting it with a BPS Lagranigan density, $\mathcal{L}_{BPS}$. The Bogomolnyi's equations are obtained when all these quadratic terms are equal to zero, $\mathcal{L} - \mathcal{L}_{BPS} = 0$. The cost for introducing this BPS Lagrangian density is additional constraint equations, which are the Euler-Lagrange equations of the BPS Lagrangian density, that must be considered in solving the Bogomolnyi's equations. As an example, a Lagrangian density of a model with k-scalar fields can be written down as,

$$\mathcal{L} - \mathcal{L}_{BPS} = \sum_{i=1}^{k} \left(\partial_\mu \phi^i - f^i(\phi^1, \ldots, \phi^N; \partial_\nu \phi^j; \vec{x}) \right) , \tag{11}$$

with $j = 1, \ldots, i-1, i+1, \ldots, k$. Setting $\mathcal{L} - \mathcal{L}_{BPS} = 0$, the Bogomolnyi's equations are given by

$$\partial_\mu \phi^i = f^i(\phi^1, \ldots, \phi^N; \partial_\nu \phi^j; \vec{x}) . \tag{12}$$

For the most of well-known cases, the BPS Lagrangian density consists of only boundary terms, or in another word its Euler-Lagrange equations are trivial [11]. In general, the BPS Lagrangian density can also contain non-boundary terms such as shown in [13] which results in BPS vortices with non-zero stress tensor in the generalized Maxwell-Higgs and Born-Infeld-Higgs models. In this article, we consider a BPS Lagrangian density only with non-boundary terms of the form[2],

$$\mathcal{L}_{BPS} = -X_0 - \frac{X_1}{r} g'(r) - \frac{X_2}{r} a'(r) - \frac{X_3}{r} A'_0(r) \,, \tag{13}$$

where $X_i \equiv X_i(g, a, A_0, N)$ with $i = 0,1,2,3$. This BPS Lagrangian is not the most BPS Lagrangian but it is be sufficient for our purpose in this article, as it will be clear in the next sections.

3.2 BPS Equations for All Effective Fields

In the BPS limit, we calculate $\mathcal{L} - \mathcal{L}_{BPS} = 0$ and solve the equation for each field one by one. We obtain,

$$g'(r) = \frac{X_1}{2rv^2 w}, \tag{14a}$$

$$a'(r) = \frac{er}{2h}\left(2eX_2 - \kappa A_0\right), \tag{14b}$$

$$A'_0(r) = \frac{\kappa(n-a) - 2eX_3}{2erh}, \tag{14c}$$

$$N'(r) = \frac{X_4}{rG}, \tag{14d}$$

and a constraint equation. This constrain function can be written in the form of a polynomial in r. Since the coefficients of this polynomial do not contain r explicitly, this constraint equation will be satisfied if each of the coefficient is zero,

$$X_0 = -\frac{(A_0\kappa - 2eX_2)^2}{8h} - A_0^2 e^2 v^2 g^2 w + V, \tag{15a}$$

$$8v^2 a^2 g^2 w = -\frac{\left(2eX_3 + (a-n)\kappa\right)^2}{e^2 h} + \frac{4X_4^2}{G} + \frac{2X_1^2}{v^2 w}. \tag{15b}$$

Besides the above constraint equation, there are other conditions that must be satisfied by the BPS Lagrangian. From the Euler-Lagrange equations of the BPS

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[2] We follow suggestions in [13] to determine explicit radial coordinate dependent for each term in the BPS Lagrangian density.

Lagrangian, we may substitute the BPS equations (14) into these equations. We then write the equations in polynomial of r and setting all the coefficients to zero. From the coefficient of r^0-term, we find

$$X_2 \equiv X_2(g, A_0, N), \tag{16a}$$

$$X_l = aY_l(g, A_0, N) + Z(g, A_0, N), \quad (l = 1, 3, 4). \tag{16b}$$

Substituting the above results into the coefficient for r^1-term, we write the obtained result as a polynomial in $a(r)$. From $a^0 r^1$-term, we obtain

$$Y_1 = \frac{1}{4eh(2eX_2 - A_0\kappa)}\left(8h^2\frac{\partial(V - A_0^2 e^2 v^2 g^2 w)}{\partial g} + \frac{\partial h}{\partial g}\left(A_0\kappa - 2eX_2\right)^2\right), \tag{17a}$$

$$Y_3 = \frac{4ev^2 g^2 A_0 hw}{A_0\kappa - 2eX_2} + \frac{\kappa}{2e}, \tag{17b}$$

$$Y_4 = \frac{1}{4eh(2eX_2 - A_0\kappa)}\left(8h^2\frac{\partial(V - A_0^2 e^2 v^2 g^2 w)}{\partial N} + \frac{\partial h}{\partial N}\left(A_0\kappa - 2eX_2\right)^2\right). \tag{18}$$

We consider a simple case where $X_0 = 0$ such that

$$X_2 = \frac{A_0\kappa}{2e} + \frac{sx_2}{e^2}\sqrt{2e^2 h(V - A_0^2 e^2 v^2 g^2 w)}, \tag{19}$$

where $sx_2 = \pm 1$ denotes the choice of signature in the solutions of X_2.

If we hold the dependency of A_0 with respect to r, then the coefficient for $a^2 r^1$ -term requires $\kappa = 0$. Therefore, we must consider $A_0(r) \equiv A_0$, where A_0 is a constant. This condition and the the Bogomolnyi's equation (14c) imply

$$Z_3 = \frac{n\kappa}{2e}, \tag{20}$$

$$V = \frac{A_0^2 e^2 v^2 g^2 w}{\kappa^2}\left(2e^2 v^2 g^2 wh + \kappa^2\right), \tag{21}$$

with a condition $\mathrm{sgn}(e) \times \mathrm{sgn}(A_0) \times \mathrm{sgn}(sx_2) \times \mathrm{sgn}(\kappa) = +1$. Substituting those functions into all the constrain equations and the equation (15b), we find the coefficient of a^2 gives

$$\frac{A_0^2e^2}{\kappa^2}\left(\frac{2v^2g^2}{G}\left(\frac{\partial(hw)}{\partial N}\right)^2+\frac{1}{g^2w}\left(\frac{\partial(g^2hw)}{\partial g}\right)^2\right)=w. \tag{22}$$

We find that the only possible solution is $Z_1=Z_4=0$ and the solution to (22) is

$$G=\frac{2v^2w\left(\dfrac{\partial(A_0eg^2hw)}{\partial N}\right)^2}{\kappa^2g^2w^2-\left(\dfrac{\partial(A_0eg^2hw)}{\partial g}\right)^2}, \tag{23}$$

where $\kappa^2g^2w^2>\left(\dfrac{\partial(A_0eg^2hw)}{\partial g}\right)^2$. The corresponding Bogomolnyi's equations are

$$\frac{a'(r)}{r}=sx_2\,2e^2v^2\sqrt{\frac{A_0^2e^2}{\kappa^2}}g^2w, \tag{24a}$$

$$g'(r)=sx_2\sqrt{\frac{A_0^2e^2}{\kappa^2}}\frac{a}{rw}\frac{\partial(g^2hw)}{\partial g}, \tag{24b}$$

$$N'(r)=sx_2\sqrt{\frac{\kappa^2}{A_0^2e^2}}\frac{a}{rw}\frac{\kappa^2g^2w^2-\left(\dfrac{\partial(A_0eg^2hw)}{\partial g}\right)^2}{\kappa^2\dfrac{\partial(g^2hw)}{\partial N}}. \tag{24c}$$

In this BPS limit, one may show that the energy density (10) becomes

$$\rho_{BPS}=-sx_2A_0^2e\sqrt{\frac{\kappa^2}{A_0^2e^2}}B+\frac{sx_2\,2v^2}{r}\sqrt{\frac{A_0^2e^2}{\kappa^2}}\frac{d(ag^2wh)}{dr}, \tag{25}$$

With B being the two-dimensional magnetic field that is defined as

$$B(r)=-\frac{1}{er}\frac{da}{dr}. \tag{26}$$

To this model, we apply the standard boundary condition for topological vortices

$$g(0)=0,\qquad a(0)=n,\qquad N(0)=N_0, \tag{27a}$$

$$g(\infty)=1, \qquad a(\infty)=0, \qquad N(\infty)=N_\infty. \tag{27b}$$

Integrating the BPS energy density (25) with respect to the boundary condition (27) throughout all space, we obtain

$$E_{BPS}=2\pi\int\rho_{BPS}r\ dr=-sx_2 2\pi n\sqrt{\frac{A_0^2\kappa^2}{e^2}}. \tag{28}$$

The negative sign indicates that positive sx_2 is related to negative winding number, and vice versa.

3.3 Numerical Calculation

For the following calculation, we do the rescaling

$$r\to\frac{r}{\kappa}, \quad N\to\frac{\kappa}{e}N, \quad A_0\to\frac{\kappa}{e}A_0, \quad v\to\frac{\kappa}{e}v. \tag{29}$$

In this model, there are three generalized coupling functions and a potential that are free to choose. Nevertheless, equation (21) and the positive definite condition of (23) reduce the number of free parameters to two. From this, we define the constraint functions to be

$$w(g,N)=\frac{\sqrt{3}(1-g^2)^2}{\sqrt{3N^2+3+3g^2-3g^4+g^6}}, \tag{30a}$$

$$h(g,N)=\frac{1}{6v^2}\frac{\sqrt{3N^2+3+3g^2-3g^4+g^6}}{g^2(1-g^2)^2}\times\left(\sqrt{3N^2+3+3g^2-3g^4+g^6}-\sqrt{3}\right), \tag{30b}$$

$$G(g,N)=\frac{3}{2v^2}\frac{N}{g^2(1-g^2)^2\sqrt{3N^2+3+3g^2-3g^4+g^6}}. \tag{30c}$$

One may check that the above definition satisfy the positive-definite condition. With this coupling functions, the potential becomes

$$V(g)=\frac{\kappa^4}{e^2}A_0^2v^2g^2(1-g^2)^2, \tag{31}$$

and the BPS equations,

$$g' = \frac{sx_2}{2}\sqrt{\frac{A_0^2}{v^4}}\frac{ag}{r}, \tag{32a}$$

$$N' = \frac{2sx_2}{\sqrt{A_0^2}}\left(1-\frac{A_0^2}{4v^4}\right)\frac{ag^2(1-g^2)^2}{rN}, \tag{32b}$$

$$\frac{a'}{r} = sx_2 2v^2\sqrt{3A_0^2}\,\frac{g^2(1-g^2)^2}{\sqrt{3N^2+3+3g^2-3g^4+g^6}}. \tag{32c}$$

From eq. (25), the energy density of this model can be written as below.

$$\rho_{BPS} = \frac{\kappa^4}{e^2}2v^2A_0^2g^2(1-g^2)^2\left(1+\frac{\sqrt{3}}{4v^4}\frac{a}{r^2\sqrt{3N^2+3+3g^2-3g^4+g^6}}\right). \tag{33}$$

The behavior of the fields near origin may give better understanding to the boundary condition for numerical analysis. By writing the solution to equations (32) as power series, we obtain

$$g(r\approx 0) = g_0 r^{n\eta_1}, \tag{34a}$$

$$N(r\approx 0) = N_0 - \frac{g_0^2\eta_2}{2N_0\eta_1}r^{2n\eta_1}, \tag{34b}$$

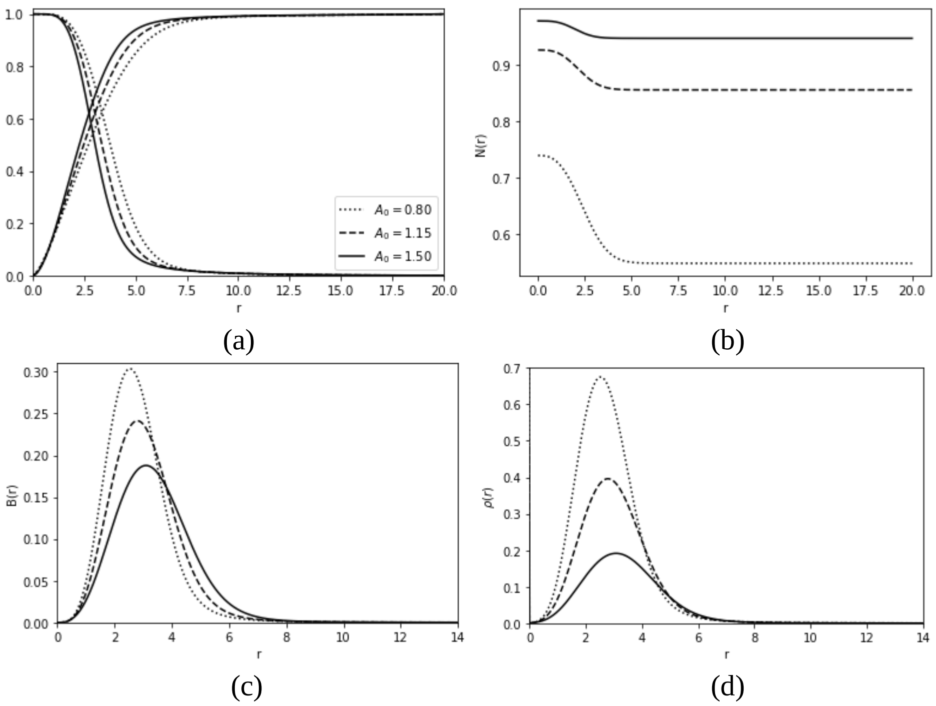
$$a(r\approx 0) = n - \frac{g_0^2\eta_3}{n\sqrt{3(N_0^2+1)}\eta_1}r^{2+2n\eta_1}. \tag{34c}$$

From the above result, we may implement the boundary condition for neutral scalar field near its origin as $N'(0)=0$.

For the numerical analysis, we present the solution for several values of the constant temporal gauge field, namely A_0 = 0.80 (dotted), A_0 = 1.15 (dash-dotted), and A_0 = 1.50 (solid). In Fig. 1.a., we present solution for the modulus of the Higgs field, $g(r)$, and the vector gauge, $a(r)$. In this calculation, we set $sx_2 = -1$ and $e=\kappa=n=v=1$. These solutions obey the standard boundary condition for topological vortices. As we increase the value of A_0, we get a steeper plot. This is due to the fact that the derivative of those functions is proportional to A_0. Numerical solution to the neutral scalar field is presented in

Fig.1.b. As we can see, it approaches constant value as we have $A_0 \to 2$. This result can be traced back to eq. (32b), that if $A_0^2 / v^4 \to 4$ then $N'(r) \to 0$. However, we cannot present the numerical solution for constant neutral scalar field due to the limitation of the numerical method that we use.

We also vary the vacuum expectation value of the Higgs field, namely $v = 0.80$ (dotted), $v = 1.00$ (dash-dotted), and $v = 1.20$ (solid), with $sx_2 = -1$ and $e = \kappa = n = A_0 = 1$. As we can see from Fig.2, the numerical results are pretty much similar to the results that we have obtained for several value of A_0. However, the solution for neutral scalar field shows the opposite trend as we increase the value of v. For the greater value of v, this solution deviates further from the constant solution. This is due to the same reason as the one that we have seen in the previous calculation. As the value of $v^4 \to 0.25$, while the other constants are unity, the solution for neutral scalar field will approach a constant solution.



(a) (b)

(c) (d)

Figure 1 (a) Numerical solution for several values of A_0 to the Higgs $g(r)$ (increasing), vector gauge field $a(r)$ (decreasing), and (b) the neutral scalar field

$N(r)$ from the BPS equations (32). In (c) the corresponding magnetic field is obtained from (26), and in (d) the energy density is presented.

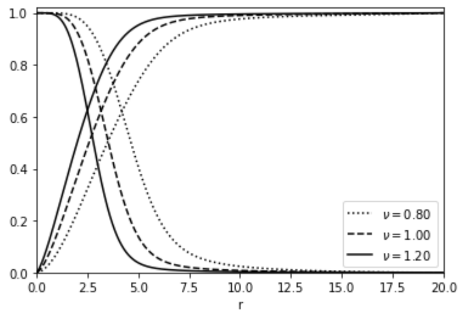
From the solution of eq. (32), we may obtain two observables namely the magnetic field and energy density. There is no particular difference for the numerical solution of the magnetic field and energy density, whether we vary the constant value of A_0 or v. However, the energy density in this solution is quite different from the one obtained before by Bazeia *et al.* [7] and Andreade *et al.* [9], that in near origin its value approaches zero. This ring-like vortices is also obtained before by Andreade *et al.* in [10]. Nevertheless, the solution that we have obtained here is still different, it has no electric field. Even so, these vortices still have magnetic charge. From (3), we may calculate the zeroth component to obtain

$$\sigma \equiv J^0 = \frac{\kappa^3}{e} 2v^2 A_0 g^2 w. \tag{35}$$

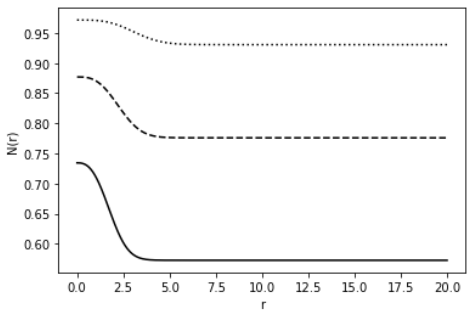
Using eq. (24a), we may integrate the charge density (35) for all space to obtain

$$Q = 2\pi \int \sigma r \, dr = -s x_2 \frac{\kappa^3}{e} 2\pi n \frac{A_0}{\sqrt{A_0^2}}. \tag{36}$$

The above result can be interpreted as the magnetic charge of this MCSH vortices.



(a)



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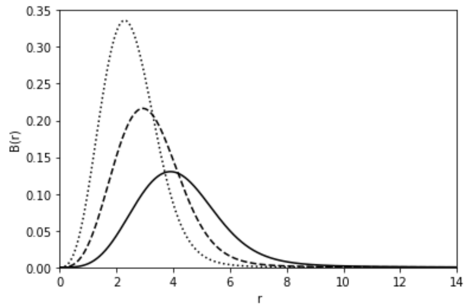
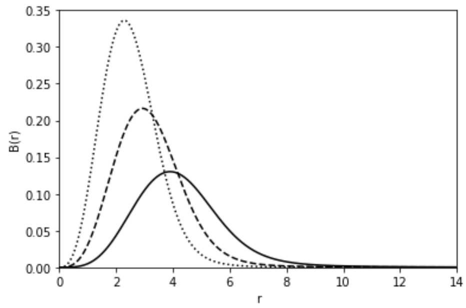


(c)



(d)

Figure 2 (a) Numerical solution for several values of v to the Higgs $g(r)$ (increasing), vector gauge field $a(r)$ (decreasing), and (b) the neutral scalar field $N(r)$ from the BPS equations (32). In (c) the corresponding magnetic field is obtained from (26), and in (d) the energy density is presented.

4 Conclusion

We have presented that topological vortex solution to the MCSH model described with Lagrangian density (1) exist. In our derivation, we hold the condition that all the involved fields are independent to one another. This leads to a conclusion that the value of the scalar gauge field is constant, thus the vortex solution that we have derived does not have electric field. However, the magnetic field for this model can be calculated using (26). From the local $U(1)$ conserved current, we may calculate the magnetic charge and we obtain that its value is proportional to the magnetic flux.

We also find that the potential depends to the generalized coupling functions $w(|\phi|, N)$ and $h(|\phi|, N)$. For numerical analysis, we introduce the explicit form for the generalized coupling functions such that they satisfy the positive definite condition. The numerical solution for $g(r)$ and $a(r)$ shows the usual behavior for the topological vortices. We do the numerical calculation for several values of A_0 and v, with the other non-varying constants are set to be unity. We obtain that the solution for neutral scalar field approaches constant solution as $A_0^2 / v^4 \to 4$. In this paper, we do not consider the solution for constant neutral scalar field since the numerical method that we use becomes ineffective.

The energy density for this model is rather different from the previous studies done by Bazeia *et al.* [7] and Andreade *et al.* in Ref. [9], that the energy density of this solution approaches zero near the origin. This ring-like solution was also obtained before by Andreade *et al.* in [10]. However, the vortex solution that they have obtained is different from our solution in a sense that their approach involves the identification between scalar gauge field and the Higgs field.

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