Probabilistic Methods for Assessing Spare Distribution Power Transformers

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Abstract. This paper addresses the challenge of determining the optimal number of spare transformers needed to maintain reliability in power distribution networks. Given the critical role of transformers in ensuring uninterrupted power supply, their failure can significantly impact the distribution system's operation. To mitigate these risks, the study explores probabilistic methods, specifically the Poisson Distribution and Markov Models, to optimize the inventory of spare transformers. The reliability criteria guide the analysis, ensuring that the system remains reliable while minimizing the financial burden of maintaining excess inventory. A case study demonstrated the comparison between two probabilistic methods using reliability criteria. The optimal number of spare transformers generated through calculations using both methods shows relatively similar results. The results highlight that while both methods are effective, the Markov Model offers a more comprehensive approach by incorporating additional parameters that accurately reflect actual conditions. This model enables utilities to balance system reliability with cost efficiency, ensuring that spare transformer inventories are maintained optimally without unnecessary expenditure. The calculation results will be compared with historical transformer failure data in the system, which are used as part of the study to validate the calculated data and demonstrate the effectiveness of both methods.c

Keywords: *power distribution networks, inventory optimization, poisson distribution, markov models*

1 Introduction

A power distribution network system carries the risk of equipment failure used in the distribution process, which can disrupt the flow of electrical energy to customers and may result in an outage. The equipment in the distribution network has varying failure rates throughout its operational life.

The distribution transformer is one of the main elements in a distribution network. Distribution systems generally operate radially, and if a distribution transformer fails, it can disrupt the power supply to customers. Two types of failures can occur in distribution transformers: minor failures, which are temporary and can be

Received ________, Revised _________, Accepted for publication __________
 ISSN: xxxx-xxxx, DOI: 10.5614/xxxx

repaired, and major failures, which are permanent and require replacing the transformer with a new one.

To maintain the reliability level of a distribution transformer system, it is crucial to have an optimal number of backup transformers in the inventory. This ensures that when a failure occurs, there is no delay in providing a replacement transformer, and the outage duration for replacing the damaged transformer can be optimized. Therefore, determining the optimal quantity of spare transformers in the warehouse is crucial to maintaining the desired level of reliability and minimizing related costs.

Several methods can be used to calculate the optimal spare transformer. The Markov model method is used to analyze the impact of variations in the number and location of spare transformers on system reliability [1]. Another research study evaluated the required spare transformers and used a mobile unit substation (MUS), which can impact the system's reliability using the Markov model [2]. A probabilistic model with Poisson distribution is used to analyze transformer reliability and calculate the optimal number of spare transformers, including the Reliability Model Criterion, Mean Time Between Failures (MTBF) Model Criterion, and Statistical Economics Model Criterion [3][4]. Another method that can be used in calculating the optimal number of spare transformers is the Monte Carlo Simulation method [5][6] and other research [7] was conducted on the optimal location of spare transformers using the Genetic Algorithm and Monte Carlo methods. The study [8] demonstrates that sharing spare transformers among distribution stations offers substantial cost savings while maintaining robust system reliability using Markov models. Compared to the previous work, this study will include comparing the calculation results obtained with historical transformer failure data, allowing for the validation of the calculations and the effectiveness of both methods

Many studies have relied on a single method to determine the optimal number of spare transformers required in a given system. However, this paper takes a different approach by evaluating the optimal number of spare transformers based on reliability criteria, utilizing a comparative analysis of two probabilistic methods: the Poisson distribution and the Markov model. While the Monte Carlo method is a powerful tool for predicting the failure rates of system components, its application to spare transformer calculations requires incorporating additional variables into a more complex formula. This study employs the Poisson distribution and Markov model to streamline the analysis and focus on practical yet robust methods. These methods are simpler to implement and effective for determining the optimal number of spare transformers needed to maintain system reliability. By comparing these two approaches, this paper aims to provide valuable insights into selecting efficient methods for spare transformer planning.

2 Reliability Criteria

2.1 Poisson Distribution

The Poisson method typically refers to the Poisson distribution in statistics. This probability model describes the occurrence of an event within a specific time interval, where the events happen independently of each other. The Poisson distribution is often used to model the number of events occurring within a fixed period when these events occur at a stable average rate. This distribution allows us to calculate the probability that an event will happen a certain number of times within a given time interval. The following equation shows the probability that a device with a failure rate λ will fail x times within the interval (0,t):

$$Px(t) = \frac{e^{-\lambda t}(\lambda t)^{x}}{x!} \qquad (1)$$

The Poisson distribution equation can be used to calculate the reliability of a system that operates with backup equipment that will be immediately activated after the primary equipment fails. In this case, the reliability equation is as follows:

$$R(t) = P_0(t) + P_1(t) \qquad (2)$$

$P_0(t)$ and $P_1(t)$Represent the probability of 0 and 1 failure occurring at time t, respectively. By combining equations 1 and 2, the following equation is obtained:

$$R(t) = e^{-\lambda t}(1 + \lambda t) \qquad (3)$$

The system's reliability, where N is the transformers in service and n represents the number of spare transformers, can be defined as the probability that no more than n failures occur within the period t. This can be written as:

$$R(t) = e^{-N\lambda t} \sum_{k=0}^{n} \frac{(N\lambda t)^{k}}{k!} \qquad (4)$$

The equation above shows that the reliability R(t) will increase as the number of spare transformers n increases. This is because having more spare transformers increases the probability that the system can continue to operate even if multiple failures occur. Each additional spare transformer increases the number of failures the system can tolerate before a complete shutdown, thereby improving the system's overall reliability.

2.2 Markov Models

This study uses the Markov model to describe the system as a series of potential conditions within a state space. The process uses specific transition probabilities to govern the shifts between these states, aiming to determine the system's reliability. The reliability value of the system used in this paper's calculations refers to the steady-state probability when the system is in a normal condition, where all transformer assets are operating, and no disturbances cause the system to enter an N-1 condition. The first Markov model in Fig. 1 illustrates the scenario for N transformers operating without spare transformers.

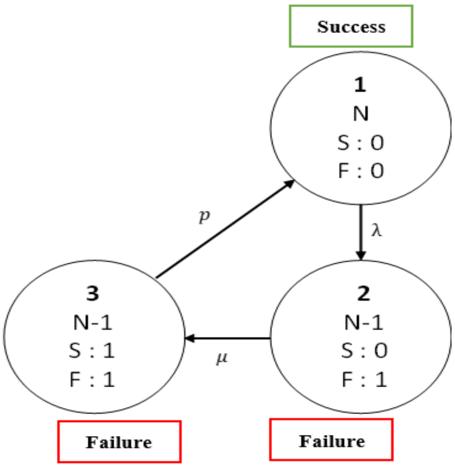

Figure 1 No Spare Markov Model.

N : Transformers in service
S : Spare transformers
F : Transformer failures
λ : Failure rate
μ : Procurement rate (New Transformer Procurement)
p : Installation Rate

Based on the model above, there are three state spaces:
State 1: This describes a typical scenario where all operating transformers function without failures.
State 2: This represents a failure condition, one transformer failure, and 0 spare transformers in the inventory.
State 3: This state represents the condition of one transformer failure, and the spare transformers are available but have not yet been installed.

The reliability probability of the system is the steady-state probability of being in State 1. The transition matrix between these state spaces can be represented as follows:

Table 1 No spare Markov Model Matrix transition.

From/To	1	2	3
1	$1-\lambda$	λ	0
2	0	$1-\mu$	μ
3	p	0	$1-p$

The steady-state probability refers to a condition in which multiplying any initial state by the transition matrix produces an output identical to the initial state.

[Initial State] x [Transition Matrix] = [Initial State] (5)

The number of state spaces will increase as the number of spare transformers in the model increases. This is due to the growing number of possible scenarios within the system. The Markov model in Fig. 2 represents a system with one spare transformer.

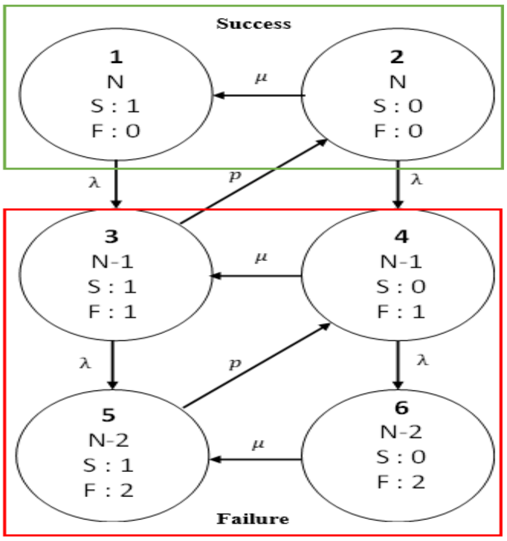

Figure 2 One Spare Markov Model.

According to the model described above, there are five possible state spaces:
State 1: N transformer in service and one spare
State 2: N transformer in service, without spare
State 3: N-1 transformer in service, one out of service, and one spare transformer.
State 4: N-1 transformer in service, one transformer out of service and without spare
State 5: N-2 transformer in service, two transformers out of service, and one spare transformer
State 6: N-2 transformer in service, two transformers out of service, without spare

As the number of spares increases, the number of possible states that can occur also increases. In the model above, normal conditions occur in States 1 and 2. Therefore, the reliability probability of the system is the sum of the steady-state probabilities of State 1 and State 2 with the transition matrix in Table II.

Table 2 No spare Markov Model Matrix transition.

From/To	1	2	3	4	5	6
1	$1-\lambda$	0	λ	0	0	0
2	μ	$1-\lambda-\mu$	0	λ	0	0
3	0	p	$1-\lambda-p$	0	λ	0
4	0	0	μ	$1-\lambda-\mu$	0	λ
5	0	0	0	p	$1-p$	0
6	0	0	0	0	μ	$1-\mu$

3 Methodology

The Poisson Distribution and Markov Model method is calculated by varying the number of spare transformers added until the desired reliability constraint is reached in the Poisson distribution and the steady state probability reaches the saturation value in the Markov model calculation. A case study uses the Transformer data assets at the PLN Distribution Unit in Kupang, East Nusa Tenggara, to obtain results that closely match actual conditions. The flowchart in this study is shown in Fig. 3.

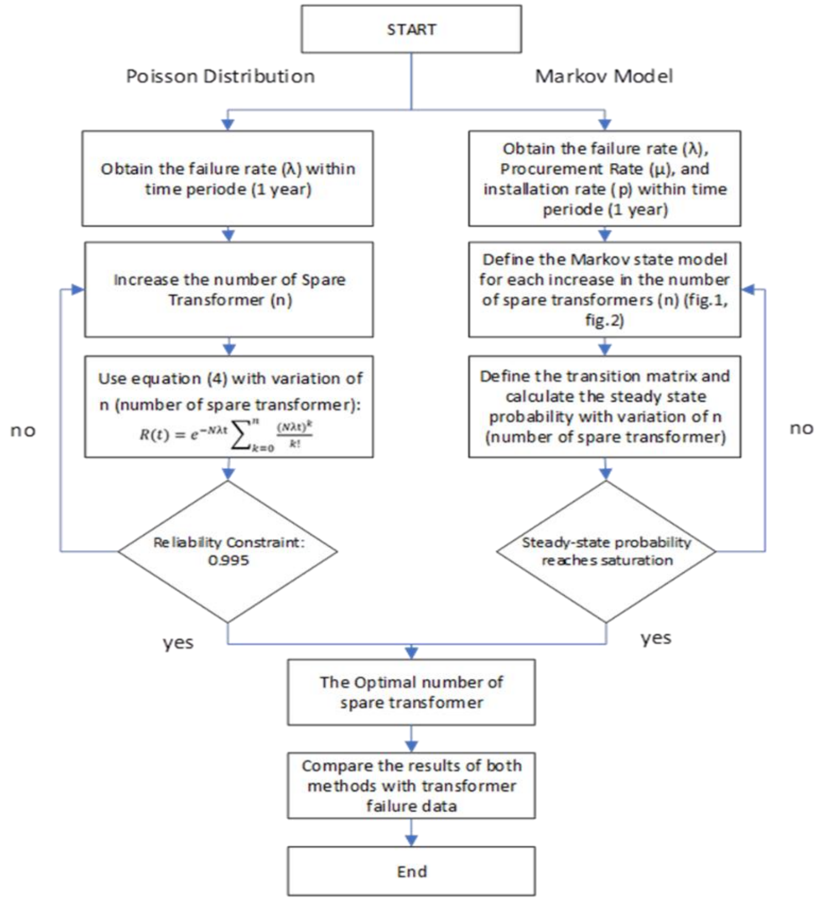

Figure 3 The Flowchart of the Calculation and Optimization Process.

The number of transformer assets and their capacities used in the study are as follows:

Table 3 Total Asset Transformer.

Capacity (KVA)	100	160	200	250
Total Assets	789	436	132	46

4 Results And Analysis

4.1 Poisson Distribution results

The distribution transformer failure rate is based on historical data of permanent faults at the Kupang Distribution Unit, with a reliability constraint of 0.995 [4], a standard for transformer reliability. In this calculation, the time required to procure a new transformer is two months or 0.167 years, including the time required for delivery and installation.

Table 4 Data for Poisson Distribution Calculation

Capacity (KVA)	100	160	200	250
Failure Rate	0.006	0.014	0.03	0.065
Procurement Rate	0.167	0.167	0.167	0.167

Using the Poisson method, the calculation is performed with equation (4), where the parameters used include historical statistical data on the number of transformer faults, expressed as the failure rate and the time required by the utility to obtain a new transformer to replace the failed one. The time used in the calculation is assumed to include the transformer replacement. This time is expressed as the procurement rate.

Table V shows the optimal results for the number of transformers needed based on Poisson Distribution method calculations. This method meets the minimum constraint for the desired reliability of 0.995.

Table 5 The Reliability calculation using Poisson Distribution

Spare	**System Reliability of Each Transformer Capacity (KVA)**			
n	**100**	**160**	**200**	**250**
1	0.797	0.736	0.856	0.988
2	0.948	0.920	0.970	**0.999**
3	0.990	0.981	**0.995**	1.000
4	**0.998**	**0.996**	0.999	1.000
5	1.000	0.999	1.000	1.000
6	1.000	1.000	1.000	1.000
7	1.000	1.000	1.000	1.000

Based on the Poisson method calculation results, the optimal number of spare transformers required is influenced by the failure rate and the time needed to procure a new transformer. As the number of spare transformers increases, the reliability will also improve. However, with the growing number of spare transformers, the required costs will also rise, and beyond the optimal number of transformers, adding more will not significantly enhance reliability.

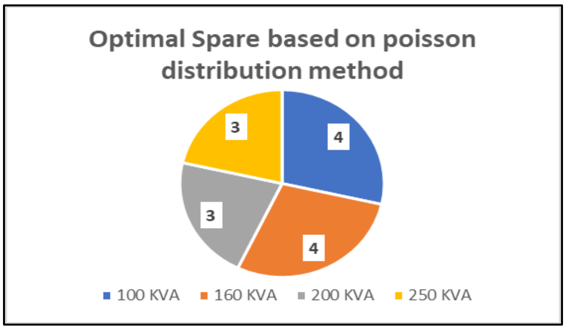

Figure 4 Optimal Spare based on Poisson distribution method.

4.2 Markov Model results

The calculation using the Markov method is performed by modeling the state space with the addition of spare transformers. With each addition of spare transformers in the model, the number of possible events can also increase, leading to an increase in the state space. The reliability value in each calculation is the sum of the probabilities of the normal condition states when reaching the steady-state condition. In the calculation using the Markov model, an additional parameter is introduced compared to the previous Poisson method: the installation rate. This installation rate serves as one of the transition probabilities between state spaces. In this calculation, the installation rate is assumed to be the time required to deliver and install a new transformer.

The calculation results using the Markov method show the probability of the system's availability as a result of a steady-state probability calculation on the normal condition in the system. Table VI shows that the reliability calculation for each transformer reaches a saturation point as the spare transformer increases, and the availability does not significantly increase with further additions.

Table 6 The Reliability calculation using Markov Model

Spare	**System Reliability of Each Transformer Capacity (KVA)**			
n	**100**	**160**	**200**	**250**
0	0.938	0.866	0.752	0.805
1	0.968	0.919	0.804	0.864
2	0.970	0.930	0.844	0.972
3	0.970	0.931	0.954	**0.975**
4	0.970	0.932	**0.974**	0.975
5	**0.998**	**0.995**	0.974	0.975

We can assume that the result is the optimal number of spare transformers needed, as shown in Fig. 5. This indicates the optimal number of spare transformers required in the system. Increasing the number of spare transformers will inevitably lead to additional investment costs, which may not significantly impact improving the system's reliability. The availability value obtained from the calculations is influenced by the assumptions regarding procurement and installation rates used in the analysis.

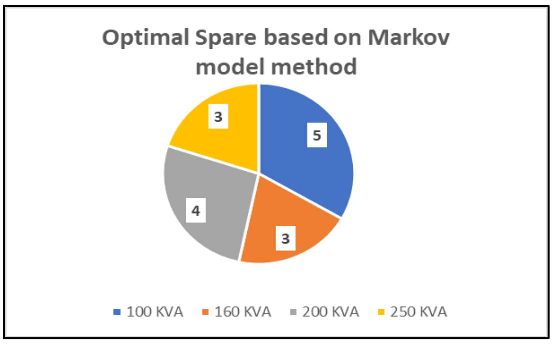

Figure 5 Optimal Spare based on Markov Model.

4.3 Comparisons between two methods

The optimization calculation results, when compared with the data on transformer failures in 2023 in the system where the research was conducted, show that both methods used produce similar and not significantly different results, as shown in Fig. 6, which indicates that the two methods are effective to calculate the optimal number of the spare transformer. In the Poisson Distribution method, the results are influenced by the failure rate and procurement rate, while in the Markov calculation, an additional parameter, the installation rate, is included. However, the Markov model calculations provide more accurate results because the modeling used in this method can represent the actual field conditions that may occur in the system, as it describes the probability of transitions from one state to another.

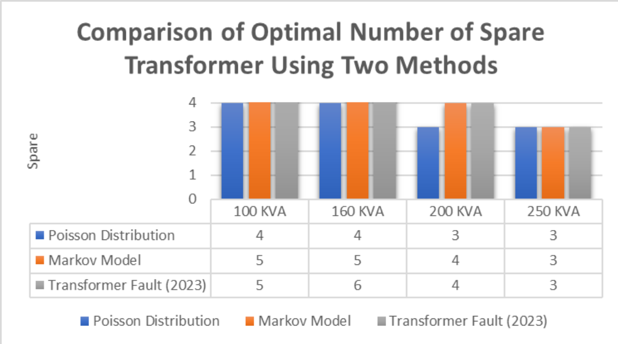

Figure 6 Comparison between the two methods.

5 Conclusion

This paper discusses a model that can determine the optimal number of spare transformers to enhance the system's reliability. The Poisson Distribution and Markov Model methods are simple and practical probabilistic approaches related to inventory optimization. The reliability model using the Markov method has advantages over the Poisson distribution method because the Markov model allows for a more accurate representation of actual conditions in the system. This enables utilities to identify the dominant parameters impacting the optimal number of spare transformers without compromising system reliability or increasing investment costs for maintaining spare transformers in inventory.

Future research can focus on optimizing these calculations by incorporating a more comprehensive range of parameters, such as the specific type and frequency of disturbances, the geographical location and accessibility of warehouse facilities that can affect the time required for transformer replacement, and the economic benefits and cost savings achieved through optimization. Considering these additional factors, the resulting models would provide more accuracy, allowing electric utility providers to make more informed, strategic decisions. This, in turn, could enhance operational efficiency, improve system reliability, and reduce overall costs in the long term. Moreover, such comprehensive optimization could help utilities better prepare for and mitigate potential risks, benefiting providers and their customers.

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