Proceedings of the 4th ITB Graduate School Conference
Innovation and Discovery for Sustainability
July 6, 2023

A Study on The Application of Virtual Crack Closure Technique and Virtual Crack Extension to Calculate Strain Energy Release Rate on Loaded Cracked Flat Plate

Enrico Samuel Rusli* & Muhammad Giri Suada

Faculty of Mechanical and Aerospace Engineering, Institut Teknologi Bandung,
Jalan Ganesa 10, Bandung 40132, Indonesia
*Email: enricorusli1107@gmail.com

Abstract. This paper compares two methods for calculating the strain energy release rate (SERR) which are the virtual crack closure technique (VCCT) and virtual crack extension (VCE). SERR is the amount of energy used for crack formation per fracture surface area. The VCCT method assumes the energy to form a crack is the same as the energy to close the crack, while the VCE method creates a virtual crack extension to calculate the change in strain energy. The study applies both VCE and VCCT to calculate the SERR for different element size, crack size, and plate thickness. The results show that the VCCT method tend to give more accurate outcome. The strain energy release rate tends to increase as the crack length increases but decreases as the plate gets thicker. Overall, this research contributes to the understanding and application of SERR calculation methods, shedding light on their accuracy and providing insights into the effects of crack length, boundary conditions, and plate thickness on SERR.

Keywords: *strain energy release rate, fracture mechanics, virtual crack extension, virtual crack closure technique.*

1 Introduction

Fracture is a phenomenon that has been studied for a long time. Fracture could happen on almost every object used by humans. As technology improves, fracture becomes a more serious issue. A small crack could lead to a very fatal incident e.g. a crack on aircraft structure, gas pipe, or nuclear reactor. These failures could cause loss of many lives, financial loss, environmental damage, and many more. Therefore, studies on crack need to be continuously developed.

Fracture mechanics is the field that studies the formation and propagation of crack in material. Crack propagates when the stress experienced by the material is greater than the cohesive strength [1], [2]. A cracked material can experience three types of displacement as shown in Figure 1 [3]. In Mode I, the crack opens in direction perpendicular to the fracture plane. In Mode II, the crack surfaces slide over each other in directions perpendicular to the crack leading edge. In Mode III, the crack surfaces slide over each other in directions parallel to the crack leading edge.

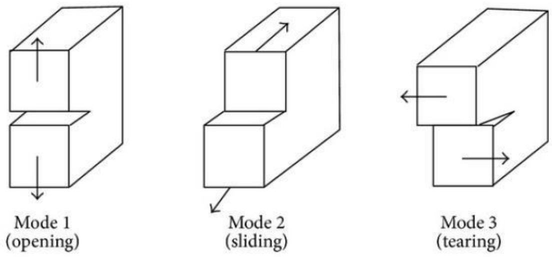


Figure 1 Crack displacement mode

Based on the First Law of Thermodynamics, a crack could form only if the process causes the total energy to decrease or remain constant [4]. The critical fracture condition happens when the energy remain constant. Therefore, the energy balance of a critical crack formation can be written as follow:

$$\frac{dE}{dA} = \frac{d\pi}{dA} + \frac{dWs}{dA} = 0 \tag{1}$$

where E is the total energy, Π is the potential energy from the internal strain energy and external forces, and W_s is the energy needed to create new surface. The above equation is also known as the Griffith energy balance. By solving the Griffith equation, the fracture stress is given by:

$$\sigma_f = \left(\frac{2E\gamma_s}{\pi a}\right)^{1/2} \tag{2}$$

where γs is the surface energy of the material and a is the half length of the fracture.

In analysing a crack, it is important to determine the crack parameters. One of the crack parameters is the strain energy release rate. The strain energy release rate is the energy that is dissipated during a formation of crack per crack surface area [5]. As a crack grows, the potential energy of the material decreases while the crack surface expands. The strain energy release rate can be defined as:

$$G = -\frac{d\Pi}{dA} \tag{3}$$

The energy release rate for a wide plate with crack in plane stress condition is given by:

$$G = -\frac{\pi\sigma^2 a}{E} \quad (4)$$

The strain energy release rates can be used to obtain other fracture parameters such as stress intensity factor, J-integral, and fracture toughness. There are several equations that can be used to calculate the strain energy release rate. However, these equations are usually very limited and could only be used for specific cases. Therefore, several approaches to calculate strain energy release rate using Finite Element Method (FEM) are developed. By using FEM, numerous variations of geometry and loading can be analyzed. FEM can also give fairly accurate results without requiring a lot of resources. FEM has been gaining popularity in the past decade due to the improvement in computer technology and the increase in accessibility of computers.

Two of the FEM methods used to analyze a crack are the Virtual Crack Extension (VCE) method and the Virtual Crack Closer Technique (VCCT). The VCE method is first developed by T. K. Hellen in 1975 [6]. Meanwhile, the original publication on VCCT dates back to 1977[7]. Since then, a lot of researches regarding both methods had been conducted [8]–[10]. These methods utilize energy differences over small changes in crack length. However, there are slight differences in both methods which will be explained in the latter chapter. In this paper, both VCE and VCCT methods will be used to calculate the strain energy release rate of a fixed plate that is given a distributed load. In this paper, all simulations will be performed using ANSYS.

2 Virtual Crack Closure Technique

VCCT is one of many ways to calculate the strain energy release rate using finite element analysis. In this method, the energy needed to extend a crack by Δa from a (Figure 2(a)) to a + Δa (Figure 2(b)) is assumed to be equal to the work required to close the crack from l to i (Figure 2(b)) [10]. For the two-dimensional case shown in Figure 2, the work ΔE necessary to close the crack can be determined as follow:

$$\Delta E = \frac{1}{2}[X_{1l}\Delta u_{2l} + Z_{1l}\Delta w_{2l}] \quad (5)$$

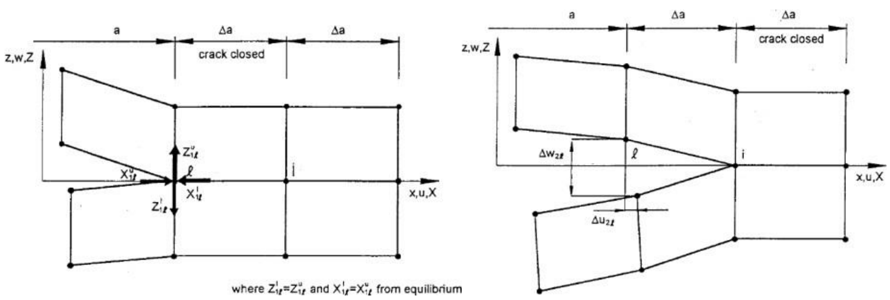


Figure 2 Virtual crack closure technique step: (a) closed crack and (b) extended crack

where X_{1l} and Z_{1l} represent the shear and opening forces at node l when the crack is closed and Δu_{2l} and Δw_{2l} are the shear and opening displacement at node l as shown in Figure 2(b). The forces are calculated at the first step while the displacements are calculated at the second step.

3 Virtual Crack Extension

Consider a crack on a 2D plate that is propagating due to Mode I loading. According to [6], the total potential energy is given by:

$$\Pi = \frac{1}{2}\{u\}^T[K]\{u\} - \{u\}^T\{b\} \tag{6}$$

where u is vector of nodal displacement, [K] is the structural stiffness matrix, and b is vector of the corresponding nodal loads. The change in potential energy then can be calculated by:

$$\delta\Pi = \frac{1}{2}\{u\}^T[\delta K]\{u\} - \{u\}^T\{\delta b\} \tag{7}$$

Assuming that the load is constant, δb will be zero. Hence, the strain energy release rate can be determined by:

$$G = -\frac{d\Pi}{dA} = -\frac{1}{2}\{u\}^T\left\{\frac{\partial K}{\partial a}\right\}\{u\} \tag{8}$$

Thus, from equation (8), a numerical procedure for calculating G can be created. The stiffness derivative for all elements except the ones containing the crack tip are zero. The crack growth can be simulated by only changing the elements around the crack tip as shown in Figure 2.3.

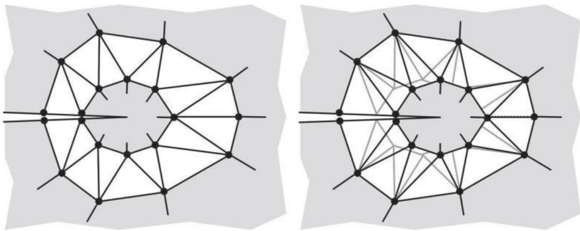


Figure 3 Virtual crack extension in finite element. Left figure shows the initial condition and right figure shows the virtual crack propagation [1]

Calculating the strain energy release rate using the VCE method is similar to the VCCT method. However, the VCE method requires only one simulation to obtain the strain energy release. The energy required to extend the crack by one element is approximately constant as the crack extend and can be calculated as follow:

$$\Delta E = \frac{1}{2}[X_i \Delta u_l + Z_i \Delta w_l] \tag{9}$$

where X_i and Z_i represent the shear and opening forces at node i and Δu_l and Δw_l are the shear and opening displacement at node l as shown in Figure 4. Therefore, the forces and displacements can be calculated in a single finite element analysis. The strain energy release rate then can be by dividing the energy ΔE by the crack surface area.

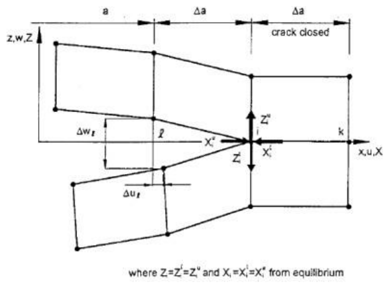


Figure 4 Virtual crack extension

4 Finite Element Model

In this paper, ANSYS 2021 R1 Student Version is used to simulate a cracked plate under an evenly distributed static load as shown in Figure 5. The plate has a width of 50 mm and thickness of 1 mm. The plate has a thin crack in the middle with a length of 20 mm. The plate uses aluminium alloy as its material. The plate is fixed on all of its sides. The plate receives distributed force with resultant of 4 Newton across its surface.

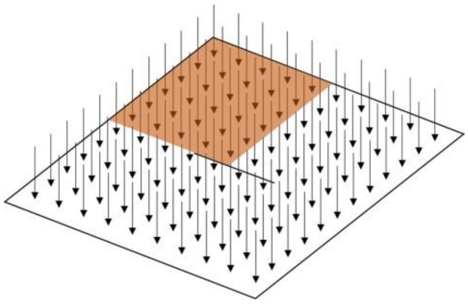


Figure 5 Model of cracked plate under distributed load

In order to simplify the simulation, only a quarter of the model will be simulated as shown in the shaded region in Figure 5. The plate uses 160x160 SHELL181 elements for its mesh. SHELL181 has 4 nodes with 6 degrees of freedom each. SHELL181 is suitable for analyzing thin to moderately-thick structures [11]. The goal of the simulation is to calculate the strain energy release rate using two methods, which are the VCCT and the VCE methods. Simulations with varying element size, crack length, and plate thickness will also be conducted.

5 Results and Discussion

For the simulation, the displacements, forces, and moments on the node around the crack tip will be extracted in order to obtain the strain energy release rate using VCE and VCCT method. The strain energy of the plate is also extracted using the available solution on ANSYS for comparison purposes.

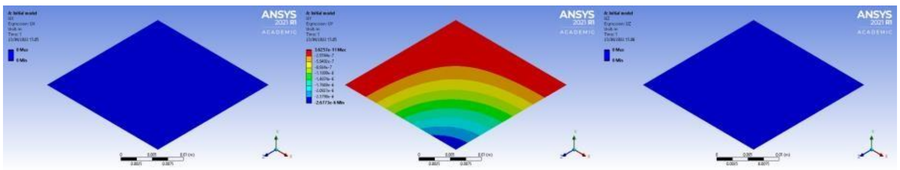


Figure 6 Translational displacements of initial model in (a) x, (b) y, and (c) z directions

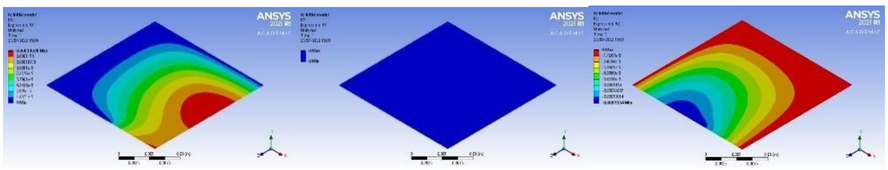


Figure 7 Rotational displacements of initial model in (a) x, (b) y, and (c) z directions

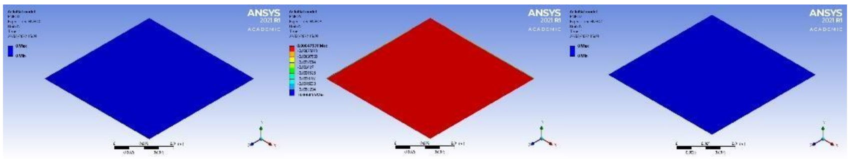


Figure 8 Nodal forces of initial model in (a) x, (b) y, and (c) z directions

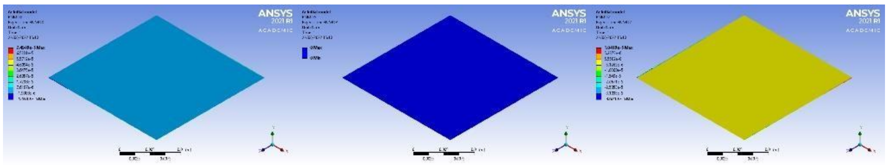
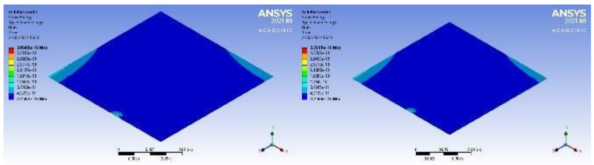


Figure 9 Nodal moments of initial model in (a) x, (b) y, and (c) z directions



(a) (b)

Figure 10 Strain energy for (a) initial and (b) extended crack

It could be observed that there is only displacement in y-direction and rotation around x-axis and z-axis axis. The symmetry boundary on the edge restrict the displacement in z-direction and rotation the rotation around x-axis and y-axis. Therefore, the only deformation that could cause the crack to propagate is the rotation around x-axis. Therefore, the VCE and VCCT method in this paper only consider the rotation and moment around x-axis.

The strain energy release rate results are presented in Table 1, indicating that both the virtual crack extension (VCE) and virtual crack closure technique (VCCT) methods yield similar outcomes. However, the VCCT method demonstrates a slightly higher level of accuracy, with a difference of only 0.0003% compared to the 0.2% difference obtained from the VCE method.

Table 1 Strain energy release rate of initial model

Method	**Value (J/m2)**
VCE	5.697328E-03
VCCT	5.686030E-03
Strain energy	5.686012E-03

5.1 Element Size Variation

In order to determine that the element size is adequate, a convergence test is conducted. The number of elements on each side of the plates is varied between 20, 40, 80, and 160. The results are shown in Figure 11.

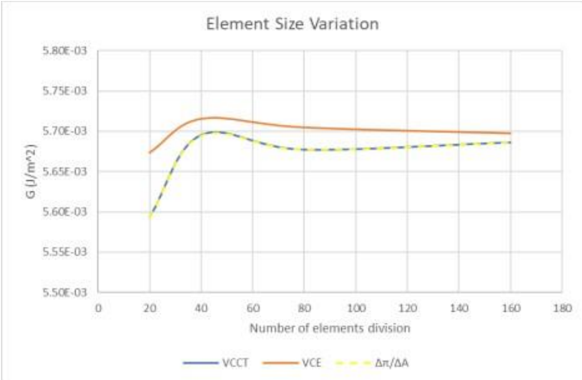


Figure 11 Strain energy release rate for different element sizes

5.2 Crack Length Variation

The length of the crack is varied to identify the relationship between the crack length and the strain energy release rate. The results are shown in Figure 12.

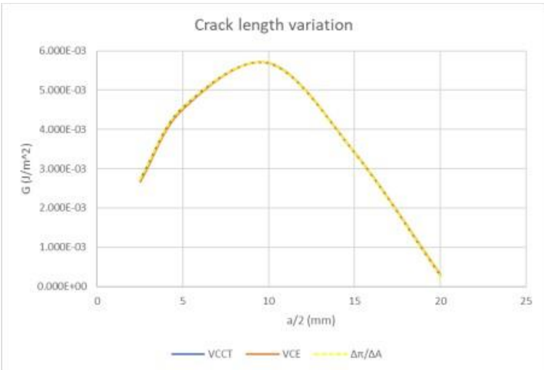


Figure 12 Strain energy release rate for different crack length

Due to the anomaly of the result, an alternative simulation is created. In his simulation, the fixed support on the edge that is parallel to the z-axis is removed. The strain energy release rate for the alternative case is shown in Figure 13.

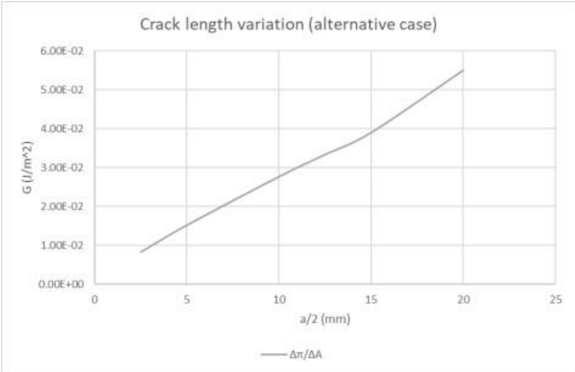


Figure 13 SERR for different crack length for alternative case

5.3 Thickness Variation

The thickness of the plate is also varied to identify the relationship between the plate thickness and the strain energy release rate. The results are shown in Figure 14.

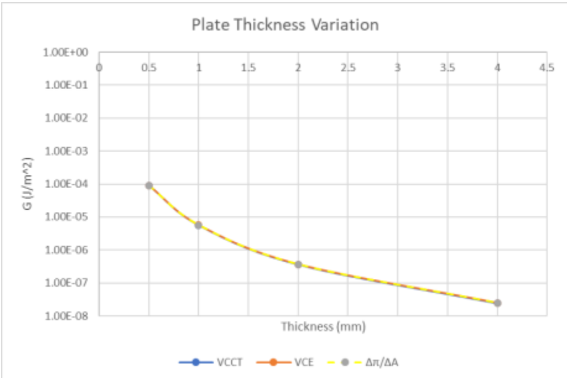


Figure 14 SERR for different plate thickness

5.4 Analysis

The initial results show that only rotational displacement around x-axis and moment around x-axis affect the propagation of crack. This rotational displacement would cause the bottom part of the crack to be expanded. Therefore, it can be concluded that the plate undergoes Mode I displacement. It is also shown in Table 1 that the strain energy release rate value obtained from VCCT method is closer to the reference strain energy difference compared to the VCE method.

Figure 11 shows that as the number of elements increases, the value of strain energy release rate is approaching a definite value. At 160x160 elements, the simulation is already converged. The figure also shows that the VCCT method yields closer value to the reference strain energy difference compared to the VCE method.

Figure 12 shows the values of strain energy release rates at different crack lengths. The results from all three methods are almost identical. The strain energy release rate initially increases as the crack length grows. However, around 10 mm half crack length, the strain energy release rate starts to decreaseas the crack length grows. This anomaly is caused by the fixed joint at the edge near the crack tip. As the crack extends, the crack tip becomes closer to the fixed boundary which limits the deformation around the crack tip. In order to prove this, an alternative simulation is conducted where the fixed boundary is removed. The result shown in Figure 13 reveals that the strain energy release rate increases as the crack grows. This result has an agreement with findings from other paper [2], [12]. This result also matches the equation (4) which shows that the strain energy release rate is proportional to the crack length.

Figure 14 shows that as the thickness of the plate increases, the strain energy release rate becomes lesser. The reduction in strain energy release rate is caused by the increase in the fracture area and also reduction in moment, displacement, and strain. This result match with the finding in other paper [12]. The results from all methods are very similar.

6 Conclusion

The author conducted calculations of the strain energy release rate using the virtual crack extension (VCE) and virtual crack closure technique (VCCT) methods. The results obtained from both methods were found to be accurate, with differences of less than 1%. The VCCT method demonstrated slightly greater accuracy, with an error of approximately 0.0003%, while the VCE method had an error of around 2%.

To ensure the validity of the simulations, various element sizes were employed, and it was demonstrated that mesh convergence was achieved with the selected element size. Simulations were also conducted with different crack lengths, revealing that the strain energy release rate increases as the crack length increases. However, due to fixed boundary conditions, the strain energy release rate begins to decrease around a 10 mm half crack length. In the absence of these boundary conditions, the strain energy release rate exhibits a linear increase with the crack length.

Additionally, simulations were performed with different plate thicknesses, showing that the strain energy release rate decreases as the plate becomes thicker. Overall, this research contributes to the understanding and application of SERR calculation methods, shedding light on their accuracy and providing insights into the effects of crack length, boundary conditions, and plate thickness on the strain energy release rate.

References

[1] T. L. Anderson, Fracture Mechanics: Fundamentals and Applications, 4th ed. CRC Press, 2017.
[2] J. L. González-Velázquez, "Linear elastic fracture mechanics," A Practical Approach to Fracture Mechanics, pp. 35–74, 2021, doi: 10.1016/B978-0-12-823020-6.00002-5.
[3] G. R. Irwin, "Analysis of Stresses and Strains Near the End of a Crack Traversing a Plate," J Appl Mech, vol. 24, no. 3, pp. 361– 364, Apr. 1957, doi: 10.1115/1.4011547.
[4] A. A. Griffith, "The Phenomena of Rupture and Flow in Solids," Philos Trans R Soc Lond, vol. 221, pp. 163–168, 1920.
[5] A. T. Zehnder, Fracture Mechanics, vol. 62. Springer Netherlands, 2012. doi: 10.1007/978-94-007-2595-9.
[6] T. K. Hellen, "On the Method of Virtual Crack Extension," Int J Numer Methods Eng, vol. 9, pp. 187–207, 1975.
[7] E. F. Rybicki and M. F. Kanninen, "A finite element calculation of stress intensity factors by a modified crack closure integral," Eng Fract Mech, vol. 9, no. 4, pp. 931–938, 1977, doi: https://doi.org/10.1016/0013-7944(77)90013-3.
[8] D. M. Parks, "The Virtual Crack Extension Method for Nonlinear Material Behaviour," Comput Methods Appl Mech Eng, vol. 12, pp. 353–364, 1977.
[9] B. Zafošnik, M. Ulbin, and J. Flašker, "Determination of Differend Fracture Modes Stress Intensity Factor with Virtual Crack Extension Method." 2000.
[10] R. Krueger, "Virtual crack closure technique: History, approach, and applications," Appl Mech Rev, vol. 57, no. 1–6, pp. 109–143, Apr. 2004, doi: 10.1115/1.1595677.
[11] "Mechanical User's Guide," ANSYS, Inc. 2021. Accessed: Apr. 25, 2023. [Online]. Available: https://ansyshelp.ansys.com/.
[12] D. P. Clausing, "Crack stability in linear elastic fracture mechanics," International Journal of Fracture Mechanics, vol. 5, no. 3, pp. 211–227, Apr. 1969, doi: 10.1007/BF00184613.