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B-$Hom(\mathbb{Z}_n, \mathbb{Z}_n)$ as B-Algebras

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Abstract. B-algebra is an algebraic structure that can be built from a group. Because the set of all integers completed by the addition operation satisfies the group property then B-algebra can be built from a group of the set of all integers completed by the addition operation. The set of all B-homomorphisms from B-algebra which is built from a group of the set of all integers modulo n can form B-algebra if it's certain properties.

Keywords: *B-algebra, group, B-homomorphism, the set of all integers modulo n.*

1 Introduction

An algebraic structure is a non-empty set that is equipped by one or more binary operations that satisfy certain properties for example rings, groups, etc [1]. Another example of an algebraic structure is B-algebra which is a class of K-algebras built from groups[2]. In 2002 J. Negger and Hee Sik Kim introduced B-algebra which is a non-empty set of X equipped by a binary operation and satisfies certain properties[3].

B-algebra is an algebraic structure that can be built from a group with 0 identity elements[1]. The set of all integers completed by the addition operation satisfies the group property. In 2021 Pramitha Shafika Wicaksono, Y. Sumanto, and Bambang Irawanto introduced B-algebra which is built from a group of the set of all integers which is a property of B-algebra if it's built from the set of all integers completed by the addition operation that is satisfied group property[1].

In the group, there is a concept of homomorphism group. Homomorphism is a mapping between two algebraic structures. Because B-algebra is an algebraic structure that can be built by groups then B-algebra also has the same concept as groups, that is B-homomorphism[1]. The set of all B-homomorphism from B-algebra A to B-algebra B is called B-$Hom(A, B)$[4]. In 2010 N.O Al-Shehri introduced on [1]$Hom(-,-)$ as B-algebra[5]. In this paper, we investigate some properties of B-$Hom(\mathbb{Z}_n, \mathbb{Z}_n)$ that is the set of all B-homomorphism from B-algebra which is built from a group of the set of all integers to B-algebra which is built from a group of the set of all integers.

2 B-Algebras

Definition 1 [6] Suppose the binary operation " $*$ " on a non-empty set A and constant 0 satisfies the following property, then *B-algebra* :

1. $x * x = 0$
2. $x * 0 = x$
3. $(x * y) * z = x * \big(z * (0 * y)\big)$

for every $x, y, z \in A$

Definition 2 [3]Let $(A;* ,0)$ is *B-algebra*, if the following conditions can be met then *B-algebra* is called *0-commutative*

$$x * (0 * y) = y * (0 * x)$$

for every $x, y \in A$

Proposition 3 [6], [7]Let $(A;\circ ,0)$ with the identity element is 0. If $x * y = x \circ y^{-1}$ for every $x, y \in A$, then $(A;* ,0)$ is *B-algebra.*

Proof. Let $x, y, z \in A$ then

a) $x * x = x \circ x^{-1} = 0$

b) $x * 0 = x \circ 0^{-1} = x \circ 0 = x$

c) $(x * y) * z = (x \circ y^{-1}) \circ z^{-1}$

$= x \circ (y^{-1} \circ z^{-1})$

$= x \circ (z \circ y)^{-1}$

$= x * (z \circ y)$

$= x * (z * y^{-1})$

$= x * \big(z * (0 * y)\big)$

So that based on definition 1 it is proven that A is *B-algebra*

Theorem 4 [4] Let $(A;\circ ,0)$ is group commutative

$$x * y = x \circ y^{-1}$$

for every $x, y \in A$, then $(A;*,0)$ is *B-algebra* and *0-commutative*

Proof. Let $x, y \in A$, then

$$\begin{aligned} x \circ y &= x \circ (y^{-1})^{-1} \\ &= x \circ (0 \circ y^{-1})^{-1} \\ &= x * (0 * y) \end{aligned}$$

And

$$\begin{aligned} y \circ x &= y \circ (x^{-1})^{-1} \\ &= y \circ (0 \circ x^{-1})^{-1} \\ &= y * (0 * x) \end{aligned}$$

So, if $x \circ y = y \circ x$ then $x * (0 * y) = y * (0 * x)$, and then based on definition 2 $(A;*,0)$ is B-algebra 0-commutative.

Definition 5 [3] Let $(A;*,0)$ is *B-algebra*, the non-empty subset N of A can be called subalgebraic *B-algebra* $(A;*,0)$ if

$$x * y \in N$$

for every $x, y \in N$

Definition 6 [3]The non-empty subset N of A is called a sub-normal algebra at *B-algebra* $(A;*,0)$ if

$$(x * a) * (y * b) \in N$$

for every $x * y, a * b \in N$

Proposition 7 [8]Let $(A;*,0)$ is *B-algebra*, then

1. $(x * z) * (y * z) = x * y$
2. $0 * (x * y) = y * x$

for every $x, y, z \in A$

3 B-Homomorphism

Definition 8 [4] Let B-algebra $(A;*,0)$ and $(B;\circ,0')$. Function $\varphi\ from\ A\ to\ B$, can be stated as $\varphi: A \to B$, then for every $x, y \in A$ called B-homomorphism if

$$\varphi(x * y) = \varphi(x) \circ \varphi(y)$$

for every $\varphi(x), \varphi(y) \in B$

Definition 9 [4] Let B-algebra $(A;*,0)$ and $(B;\circ,0')$. Function $\theta: A \to B$ called B-homomorphism trivial if $\theta(x) = 0'$, for every $x \in A$

Definition 10 [4] Let $(A;*,0)$ and $(B;\circ,0')$ is B-algebra then the set of all B-homomorphism form B-algebra $(A;*,0)$ to B-algebra $(B;\circ,0')$ can be written as

B-homomorphism (A, B)

Definition 11 [4] Let $(A;*,0)$ and $(B;\circ,0')$ is B-algebra and $\varphi \in B - \mathrm{hom}(A, B)$ then apply:

$$\varphi(0) = 0'$$

$$\varphi(0 * x) = 0' \circ \varphi(x)$$

for every $x \in A$

4 B-Algebra $(\mathbb{Z}_n;*,[0]_n)$ Defined from The Group $(\mathbb{Z}_n, +_n)$

Theorem 12. [1] Let $(\mathbb{Z}_n, +_n)$ be group, defined binary operation " $*$ " in $\mathbb{Z}_n$ with $[x_n] * [y_n] = [x]_n -_n [y_n]$, for every $[x_n],[y_n] \in \mathbb{Z}_n$, then $(\mathbb{Z}_n;*,[0]_n)$ is B-algebra.

Theorem 13. [1] Let $(\mathbb{Z}_n, +_n)$ be group and $(\mathbb{Z}_n;*,[0]_n)$ be B-algebra which is defined with $[x]_n * [y]_n = [x]_n -_n [y]_n$, for every $[x_n],[y_n] \in \mathbb{Z}_n$, then $(\mathbb{Z}_n;*,[0]_n)$ is 0-commutative B-algebra.

Theorem 14. [1] Let $(\mathbb{Z}_n, +_n)$ be group and function $f: \mathbb{Z}_n \to \mathbb{Z}_n$, function f is a group homomorphism if and only if it exists $[k]_n \in \mathbb{Z}_n$ so that $f([m]_n) = [k]_m [m]_n$ for every $[m]_n \in \mathbb{Z}_n$.

Theorem 15. [1] Let $(\mathbb{Z}_n, +_n)$ be group and group homomorphism $f: \mathbb{Z}_n \to \mathbb{Z}_n$, if $(\mathbb{Z}_n;*,[0]_n)$ is B-algebra which is defined with $[x]_n * [y]_n = [x]_n -_n [y]_n$, for every $[x_n],[y_n] \in \mathbb{Z}_n$, then $f: \mathbb{Z}_n \to \mathbb{Z}_n$ is also B-homomorphism.

Theorem 16 [1] Let $(\mathbb{Z}_n;*,[0]_n)$ be B-algebra and B-Hom$(\mathbb{Z}_n, \mathbb{Z}_n)$ is the set of all B-homomorphism from B-algebra $(\mathbb{Z}_n;*,[0]_n)$ to B-algebra $(\mathbb{Z}_n;*,[0]_n)$. If in B-Hom$(\mathbb{Z}_n, \mathbb{Z}_n)$defined binary operation " $\circledast$ " with

$$(f \circledast g)([x]_n) = f([x]_n) * g([x]_n)$$

For every $[x]_n \in \mathbb{Z}_n$ with $\theta([x]_n) = [0]_n$ for every $[x]_n \in \mathbb{Z}_n$, then B-Hom$(\mathbb{Z}_n, \mathbb{Z}_n);\circledast, \theta)$ is B-algebra.

5 B-Hom$(\mathbb{Z}_{\boldsymbol{n}}, \mathbb{Z}_{\boldsymbol{n}})$ as B-algebra

Definition 17. Given $(\mathbb{Z}_n;*, [0]_n)$ is a B-algebra and B-Hom$(\mathbb{Z}_n, \mathbb{Z}_n)$ is the set of all B-homomorfisma from B-algebra$(\mathbb{Z}_n;*, [0]_n)$ to B-algebra$(\mathbb{Z}_n;*, [0]_n)$. In B-Hom$(\mathbb{Z}_n, \mathbb{Z}_n)$ defined operation " $\circledast$ " with

$$(f \circledast g)([a]_n) = f([a]_n) * g([a]_n)$$

for every $f, g \in$ B-Hom$(\mathbb{Z}_n, \mathbb{Z}_n)$ and $\theta([a]_n) = [0]_n$ for every $[a]_n \in \mathbb{Z}_n$

Theorem 18. Given $(\mathbb{Z}_n;*, [0]_n)$ is a B-algebra, then (B-Hom$(\mathbb{Z}_{\boldsymbol{n}}, \mathbb{Z}_{\boldsymbol{n}}), \theta)$ is B-algebra 0-commutative

Proof:

Based on Theorem 19, since $(\mathbb{Z}_n;*, [0]_n)$ is B-algebra then (B-Hom$(\mathbb{Z}_{\boldsymbol{n}}, \mathbb{Z}_{\boldsymbol{n}});\circledast, \theta)$.

We will prove that (B-Hom$(\mathbb{Z}_{\boldsymbol{n}}, \mathbb{Z}_{\boldsymbol{n}});\circledast, \theta)$ is B-algebra 0-commutative.

Based on Theorem 16, $(\mathbb{Z}_n;*, [0]_n)$ is B-algebra 0-commutative

Let $f, g \in$ Hom$(\mathbb{Z}_{\boldsymbol{n}}, \mathbb{Z}_{\boldsymbol{n}})$ and $[a]_n \in \mathbb{Z}_n$ then

$$\begin{aligned}\big(f \circledast (\theta \circledast g)\big)([a]_n) &= f([a]_n * (\theta([a]_n) * g([a]_n)) \\ &= f([a]_n) * ([0]_n * g([a]_n)) \\ &= g([a]_n * ([0]_n * f([a]_n)) \\ &= g([a]_n * (\theta([a]_n) * f([a]_n)) \\ &= (g \circledast (\theta \circledast f))([a]_n)\end{aligned}$$

so that $f \circledast (\theta \circledast g) = g \circledast (\theta \circledast f)$. Based on definition 3, (B-Hom$(\mathbb{Z}_{\boldsymbol{n}}, \mathbb{Z}_{\boldsymbol{n}});\circledast, \theta)$ is B-algebra 0-comutative

Example 19. Given $(\mathbb{Z}_3, +_3)$, a group under operation "$+_3$" which is additional operation of modulo 3 shown in the following table.

Table 1 Definition Table of Operation "$+_3$" in $\mathbb{Z}_3$

$+_3$	$[0]_3$	$[1]_3$	$[2]_3$
$[0]_3$	$[0]_3$	$[1]_3$	$[2]_3$
$[1]_3$	$[1]_3$	$[2]_3$	$[0]_3$
$[2]_3$	$[2]_3$	$[0]_3$	$[1]_3$

Given a biner operation " $*$ " defined as

$$[x]_3 + [y]_3 = [x]_3 -_3 [y]_3$$

For every $[x]_3, [y]_3 \in \mathbb{Z}_3$ that is shown in the following table.

Table 2 Definition Table of Operation " $*$ " in $\mathbb{Z}_3$

$+_3$	$[0]_3$	$[1]_3$	$[2]_3$
$[0]_3$	$[0]_3$	$[2]_3$	$[1]_3$
$[1]_3$	$[1]_3$	$[0]_3$	$[2]_3$
$[2]_3$	$[2]_3$	$[1]_3$	$[0]_3$

0-comutative. The functions $\theta, I, \mu: \mathbb{Z}_3 \to \mathbb{Z}_3$ respectively are $\theta([x]_3) = [0]_3$, $I([x]_3) = [x]_3$ and $\mu([x]_3 = [2]_3[x]_3$ for every $[x]_3 \in \mathbb{Z}_3$ then B-Hom$(\mathbb{Z}_3, \mathbb{Z}_3)$ is B-algebra 0-comutative.

The function $\theta: \mathbb{Z}_3 \to \mathbb{Z}_3$ with $\theta([x]_3) = [0]_3$, for every $[x]_3 \in \mathbb{Z}_3$ shown in the Picture 1.

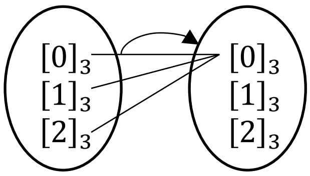

Figure 1 The Function $\theta: \mathbb{Z}_3 \to \mathbb{Z}_3$

The function $I: \mathbb{Z}_3 \rightarrow \mathbb{Z}_3$ with $I([x]_3 = [x]_3$ for every $[x]_3 \in \mathbb{Z}_3$ shown in the Picture 2.

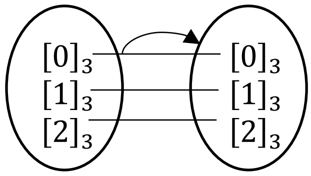

Figure 2 The function $\theta: \mathbb{Z}_3 \rightarrow \mathbb{Z}_3$

The function $\mu: \mathbb{Z}_3 \rightarrow \mathbb{Z}_3$ with $\mu([x]_3 = [2]_3[x]_3$, for every $[x]_3 \in \mathbb{Z}_3$ shown in the Picture 3.

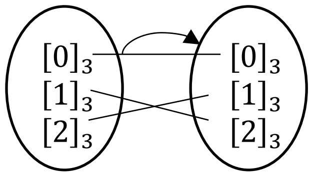

Figure 3 The function $\mu: \mathbb{Z}_3 \rightarrow \mathbb{Z}_3$

Based on Theorem 18, since $(\mathbb{Z}_3; *, [0]_3)$ is B-algebra 0-comutative then (B-Hom$(\mathbb{Z}_2, \mathbb{Z}_2)$; $\circledast, \theta)$ is B-algebra 0-commutative.

The operation " $\circledast$ " in B-Hom$(\mathbb{Z}_3, \mathbb{Z}_3)$ shown in the following table.

Table 3 Definition Table of Operation " $\circledast$ " in B-Hom$(\mathbb{Z}_3, \mathbb{Z}_3)$

$\circledast$	θ	I	μ
θ	θ	μ	I
I	I	θ	μ
μ	μ	I	θ

Definition 20 [1] Let M and $\circledast$ be subsets of X and $Hom(X, Y)$, respectively. We define orthogonal subsets $M^{\perp}$ and $\circledast^{\perp}$ of M and $\circledast$, respectively, by

$M^{\perp} = \{f \in Hom(X, Y) | f(x) = 0$, for all $x \in M\}$

and

$\circledast^{\perp} = \{x \in X | f(x) = 0$, for all $f \in Hom(X, Y)\}$

Theorem 21. Given B-algebra $(\mathbb{Z}_n; *, [0]_n)$. Let $M \subseteq \mathbb{Z}_n$ and $\theta \subseteq$ B-Hom$(\mathbb{Z}_n, \mathbb{Z}_n)$, then $M^{\perp}$ and $\theta^{\perp}$ respectively are normal subalgebra of (B-Hom$(\mathbb{Z}_n, \mathbb{Z}_n)$; $\circledast, \theta$) and $(\mathbb{Z}_n; *, [0]_n)$.

Proof:

Since for every $[x]_n * [y]_n, [a]_n * [b]_n \in M$ applies

$$([x]_n * [a]_n) * ([y]_n * [b]_n) \in M$$

Based on Definition 6, M is normal subalgebra in B-algebra $(\mathbb{Z}_3; *, [0]_n)$.

We will show that $M^{\perp}$ is normal subalgebra of (B-Hom$(\mathbb{Z}_n, \mathbb{Z}_n)$; $\circledast, \theta$)

Let $f \circledast g, h \circledast j \in M^{\perp}$ and since $M^{\perp} = \{$f $\in$ B-Hom$(\mathbb{Z}_n, \mathbb{Z}_n) | f([a]_n = [0]_n$, for every $[a]_n \in M\}$, then

$(f \circledast g)([a]_n) = [0]_n$, for every $[a]_n \in M$

$(h \circledast j)([a]_n) = [0]_n$, for every $[a]_n \in M$

Based on Theorem 18, (B-Hom$(\mathbb{Z}_n, \mathbb{Z}_n)$; $\circledast, \theta$) is B-algebra 0-commutative then

$$\begin{aligned}
\big((f \circledast h) \circledast (g \circledast j)\big)([a]_n) &= \Big(f \circledast \big((g \circledast j \circledast (\theta * h))\big)\Big)([a]_n) \\
&= \Big(f \circledast \Big(g \circledast \big((\theta \circledast h) \circledast (\theta \circledast j)\big)\Big)\Big)([a]_n) \\
&= \Bigg(f \circledast \Big(g \circledast \Big(j \circledast \big(\theta \circledast (\theta \circledast h)\big)\Big)\Big)\Bigg)([a]_n) \\
&= \Big(f \circledast \big(g \circledast (j \circledast h)\big)\Big)([a]_n) \\
&= \Bigg(f \circledast \Big(g \circledast \Big(j \circledast \big(\theta \circledast (\theta \circledast h)\big)\Big)\Big)\Bigg)([a]_n) \\
&= \big(f \circledast \big(\theta \circledast (g \circledast h)\big) \circledast j\big)([a]_n) \\
&= \big((j \circledast h) \circledast (\theta \circledast f)\big) \circledast g)\,([a]_n) \\
&= \Big((j \circledast h) \circledast \Big(g \circledast \big(\theta \circledast (\theta \circledast f)\big)\Big)\Big)([a]_n) \\
&= \big((j \circledast h) \circledast (g \circledast f)\big)([a]_n) \\
&= \Big(\Big(\big(\theta \circledast (h \circledast j)\big) \circledast \big(\theta \circledast (f \circledast g)\big)\Big)\Big)([a]_n)
\end{aligned}$$

$$= \Big(\theta([a]_n) * \big(h([a]_n) * j([a]_n)\big)$$
$$* \Big(\theta([a]_n) * \big(f([a]_n) * j([a]_n)\big)\Big)\Big)$$
$$= ([0]_n * [0]_n) * ([0]_n * [0]_n)$$
$$= [0]_n * [0]_n$$
$$= [0]_n$$

Since $\big((f \circledast h) \circledast (g \circledast j)\big)([a]_n) = [0]_n$ so that $(f \circledast h) \circledast (g \circledast j) \in M^{\perp}$ then based on Definition 6, $M^{\perp}$ is normal subalgebra of (B-Hom$(\mathbb{Z}_n, \mathbb{Z}_n)$; $\circledast, \theta)$.

Next, we will show that $\theta^{\perp}$ is normal subalgebra of $(\mathbb{Z}_n; *, [0]_n)$.

Let $[x]_n * [y]_n, [a]_n * [b]_n \in \theta^{\perp}$ then

$$f([x]_n * [y]_n) = [0]_n$$
$$g([a]_n * [b]_n) = [0]_n$$

so that

$$f\big(([x]_n * [a]_n) * ([y]_n * [b]_n)\big) = f\Big([x]_n * \big(([y]_n * [b]_n) * ([0]_n * [a]_n)\big)\Big)$$
$$= f\Big([x]_n$$
$$* \Big([y]_n * \big(([0]_n * [a]_n) * ([0]_n * [b]_n)\big)\Big)\Big)$$
$$= f([x]_n * \Big(\big([y]_n * ([b]_n * [a]_n)\big)\Big)$$
$$= f\Big(\Big([x]_n * \big([0]_n * ([b]_n * [a]_n)\big)\Big) * [y]_n\Big)$$
$$= f\Big(\big(([b]_n * [a]_n) * ([0]_n * [x]_n)\big) * [y]_n\Big)$$
$$= f\Big(([b]_n * [a]_n)$$
$$* \big([y]_n * [0]_n * ([0]_n * [x]_n)\big)\Big)$$
$$= f\Big(\big([b]_n * [a]_n * ([y]_n * [x]_n)\big)\Big)$$
$$= f\Big(\big([0]_n * ([a]_n * [b]_n)\big)$$
$$* \big([0]_n * ([x]_n * [y]_n)\big)\Big)$$
$$= \big(f[0]_n * f([a]_n * [b]_n)\big) * (f([0])n)$$
$$* f([x]_n * [y]_n))$$
$$= ([0]_n * [0]_n) * ([0]_n * [0]_n)$$
$$= [0]_n * [0]_n$$
$$= [0]_n$$

Since $f\big(([x]_n * [a]_n) * ([y]_n * [b]_n)\big) = [0]_n$ so that $f\big(([x]_n * [a]_n) * ([y]_n * [b]_n)\big) \in \theta^{\perp}$ then based on Definition 6, $\theta^{\perp}$ is normal subalgebra of $(\mathbb{Z}_n; *, [0]_n)$.

Theorem 22 Let B-algebra $(\mathbb{Z}_n;*, [0]_0)$. For example $M \subseteq \mathbb{Z}_n$ and $\theta \subseteq B - Hom(\mathbb{Z}_n, \mathbb{Z}_n)$, then $M^{\perp}$ and $\theta^{\perp}$ successively is a subalgebra of $(B - Hom\ (\mathbb{Z}_n, \mathbb{Z}_n;\circledast, \theta)$ and $(\mathbb{Z}_n;*, [0]_n$

Proof. Because for every $[x]_n, [y]_n \in M$ apply

$$[x]_n * [y]_n \in M$$

Then by definition, M subalgebra on (B-algebra $(\mathbb{Z}_n, \mathbb{Z}_n);\circledast, \theta)$.

Let $f, g \in M^{\perp}$, because $M^{\perp} = \xi f \in B - Hom(\mathbb{Z}_n, \mathbb{Z}_n) | f \circledast = [0]_n, \forall x \in M)$

Then:

$$f([a]_n) = [0]_n, \forall [a]_n \in M$$
$$g([a]_n) = [0]_n, \forall [a]_n \in M$$

So that

$$\begin{aligned}(f \circledast g)([a]_n) &= f([a]_n) * g([a]_n) && \text{Definition 17}\\ &= [0]_n * [0]_n \\ &= [0]_n && \text{Definition 1 No 1}\end{aligned}$$

Because $(f \circledast g)([a]_n) = [0]_n$ then $f \circledast g \in M^{\perp}$ then by definition 5 $M^{\perp}$ subalgebra from $(B - Hom\ (\mathbb{Z}_n, \mathbb{Z}_n);\circledast, \theta)$

Showed $\theta^{\perp}$ is subalgebra B-algebra $(\mathbb{Z}_n; *, [0]_n$

Let $[x]_n, [y]_n \in \theta^{\perp}$ then

$$f([x]_n) = [0]_n, \qquad \forall [x]_n \in \theta$$
$$f([y]_n) = [0]_n, \qquad \forall [y]_n \in \theta$$

So that

$$\begin{aligned}f([x]_n * [y]_n) &= f([x]_n) * f([y]_n)\\ &= [0]_n * [0]_n\\ &= [0]_n && \text{Definition 1 No 1}\end{aligned}$$

Because $f([x]_n * [y]_n) = [0]_n$ then $f([x]_n * [y]_n \in \theta^{\perp}$ then by definition 5 $\theta^{\perp}$ subalgebra from *B-algebra* $(\mathbb{Z}_n; *, [0]_n)$

Preposition 23 Given group $(\mathbb{Z}_n, +_n)$ and $(\mathbb{Z}_n; *, [0]_n)$ is *B-algebra* defined with

$$[x]_n * [y]_n = [x]_n - n[y]_n$$

For every $[x]_n, [y]_n \in \mathbb{Z}_n$, if $n \leq 2, n \in \mathbb{N}$ then $(\mathbb{Z}_n; *, [0]_n)$ is *B-algebra* associative.

6 Conclusion

B-algebra is an algebraic structure that can be built from a group. B-algebra built from the $\mathbb{Z}_n$ group equipped with binary operations, namely addition and its properties. The set of all B-homomorphisms of B-algebra which is constructed from the set of all integers modulo n can form a B-algebra if it has certain properties. B-algebra is an algebraic structure that can be built from a group with 0 identity elements, within a group, there is the concept of group homomorphism. Because B-algebra is an algebraic structure that can be built by groups, B-algebra also has the same concept as groups, namely B-homomorphism. Based on the definitions and theorems that have been proven, it can be stated that some of the properties of B-Hom$(\mathbb{Z}_n, \mathbb{Z}_n)$ which are the set of all B-homomorphisms of B-algebra which are constructed from groups of the set of all integers become B-algebras which are constructed from groups from the set of all integers that satisfy certain properties.

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