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Deep Learning for Data-Driven Turbulence Modeling in Flow over Periodic Hills

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Abstract. Data-driven method has gained rapid growth in recent years. It is driven by the rise of big data in various fields. Nowadays, Deep learning is the most famous data-driven method used in wide range of applications such as in fluid mechanics. Turbulence modeling is an unsolved problem in fluid mechanics. Reynolds-averaged Navier-Stokes (RANS) is the most popular method for turbulence modeling in real-world problems. The objective of RANS turbulence modeling is to relate the Reynolds stress with the mean flow properties. The weakness of the RANS model has driven the research to develop another approach. The application of deep learning in turbulence modeling has shown promising results in recent years. In this work, deep learning is used to develop a model for turbulence closure modeling. The performance of this model is compared with RANS $k-\omega$ model as the classical turbulence model. From the results of this work, it is shown that the neural network model proposed by the author could give better performance on giving the closure relation for turbulent flow over periodic hills which gives 57% RMSE improvement from the RANS model and could capture the separation phenomenon when RANS model is struggling.

Keywords: *deep learning; turbulence model; RANS; neural network; Reynolds stress.*

1 Introduction

Fluid mechanics is one of the most highly sought-after applied science studies. The study of fluid mechanics is essential in various types of applications such as flow within wind turbines, combustion chambers, racecar aerodynamics, etc. One of the biggest problems that has not been completely solved in fluid mechanics is turbulence. Turbulence is a state of fluid flow where it is very sensitive to any change in its condition and causes a fluctuating phenomenon in the flow properties. Direct numerical solution (DNS) is the most accurate approach to obtain the complete solution of turbulent flow. Unfortunately, this approach is limited by computational resources and it is impossible to conduct a DNS for real-world fluid mechanics problems. Even with Moore's law, we are likely several decades away from resolving all scales in configurations of industrial interest [1]. The most common method in solving turbulent flow is Reynolds-averaged Navier-Stokes (RANS). This method needs a turbulence model to enclose the system of governing equations. Classical RANS turbulence models such as the linear Eddy-viscosity model (LEVM) are struggling to model Reynolds stress in several flow configurations with curvature and separation [2, 3] such as flow over periodic hills.

The rise of machine learning applications in various real-world problems led to the application of this method in developing data-driven turbulence model [4, 5, 6]. The application of machine learning in turbulence modeling has been an interesting research topic and an active area of research in recent years [7]. One important work is done by Ling et al. [8] which they used neural network that embeds physical invariance to calculate the non-dimensional anisotropy Reynolds stress tensor. The architecture proposed by Ling et al. is well-known as tensor basis neural network (TBNN). Similar work was done by Kandoorp and Dwight [9] which used the random forest model. These two works used the pope integrity basis [10] which guarantees that the model preserves Galilean invariance. Rui Fang et al. [11] used neural network model to predict non-dimensional anisotropy stress tensor in turbulent channel flow. These three works show promising results that could give a better turbulence model than RANS. In this work, we attempted to develop a neural-network-based turbulence model for solving turbulent flow over periodic hills.

2 Problem Statement

Flow over periodic hills is a two-dimensional flow problem commonly used as a benchmark for turbulence modeling. This flow is a challenging flow problem for Reynolds-averaged Navier-Stokes (RANS) method since it features a flow separation phenomenon. The direct numerical simulation (DNS) of flow over periodic hills was performed by Xiao et al. [12]. This DNS dataset can be used to develop a data-driven turbulence model using neural network. The dataset consists of five cases, parameterized by the variation of steepness ratio (α). The geometry of flow over periodic hills is given in Figure 1.

Figure 1 Geometry of flow over periodic hills

Applying the Reynolds-averaging technique to the incompressible Navier–Stokes equations yields the RANS equations,

$$\frac{\partial \langle u_i \rangle}{\partial x_i} = 0 \quad (1)$$

$$\frac{\partial \langle u_i \rangle}{\partial t} + \langle u_k \rangle \frac{\partial \langle u_i \rangle}{\partial x_k} = -\frac{1}{\rho}\frac{\partial \langle p \rangle}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\nu\left(\frac{\partial \langle u_i \rangle}{\partial x_j} + \frac{\partial \langle u_j \rangle}{\partial x_i}\right) - \langle u_i' u_j' \rangle\right] \quad (2)$$

The RANS equations yield additional unknown terms $\langle u_i' u_j' \rangle$ which known as the Reynolds stress. To enclose the RANS equations, we used $k - \omega$ model proposed by Wilcox [13] as the baseline model. In this model, two additional transport equations are solved to compute the turbulent kinetic energy and specific rate of dissipation. The two additional transport equations are given below,

$$\frac{\partial(\rho k)}{\partial t} + \frac{\partial(\rho u_j k)}{\partial x_j} = P - \beta^* \rho \omega k + \frac{\partial}{\partial x_j}\left[\left(\mu + \sigma_k \frac{\rho k}{\omega}\right)\frac{\partial k}{\partial x_j}\right] \quad (3)$$

$$\frac{\partial(\rho \omega)}{\partial t} + \frac{\partial(\rho u_j \omega)}{\partial x_j} = \frac{\gamma \omega}{k} P - \beta \rho \omega^2 + \frac{\partial}{\partial x_j}\left[\left(\mu + \sigma_\omega \frac{\rho k}{\omega}\right)\frac{\partial \omega}{\partial x_j}\right] \quad (4)$$

To calculate the Reynolds stress in $k - \omega$ model, we use the equation below,

$$\nu_t = \frac{k}{\omega} \quad (5)$$

$$\tau_{ij} = -\nu_t\left(\frac{\partial \langle u_i \rangle}{\partial x_j} + \frac{\partial \langle u_j \rangle}{\partial x_i}\right) + \frac{2}{3} k \delta_{ij} \quad (6)$$

The accuracy of $k - \omega$ model as the baseline model will be compared with the neural network-based turbulence model developed in this work.

3 Methodology

Deep learning or neural network attempts to find the optimal weights (*w)* and biases (*b)* for a given input feature *(x).* This could be done by performing the optimization task to minimize the residual error between the true output (*y*) and model output ($F(w; b; x)$) below,

$$w; b = argmin \, \|y - F(w; b; x)\|^2 \quad (7)$$

To develop the model, we use the DNS data as the true output and the input features from the RANS mean flow data. The minimization is performed using gradient descent technique by using the Adam optimizer [14]. In this work, the neural network architecture that is used to develop the model is the feed forward neural network (FFNN) model. In the FFNN model, neural network is used to predict Reynolds stress (τ_{ij}) based on output results from RANS solver using $k - \omega$ turbulence model. The flowchart of FFNN model framework is given in Figure 2.

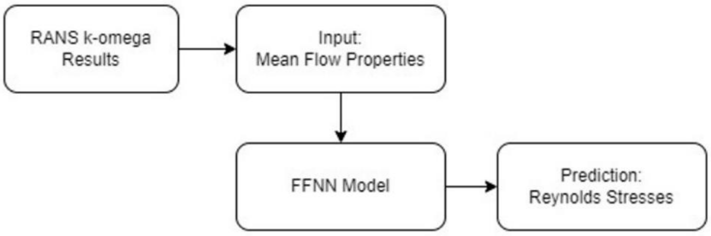

Figure 2 Flowchart of FFNN model framework.

In RANS turbulence modeling, we try to relate the Reynolds stress with the mean flow properties. The input features are mean velocity gradient, turbulent kinetic energy gradient and mean pressure gradient.

$$\tau = f(\nabla U, \nabla k, \nabla p) \tag{7}$$

These features are chosen since those are the flow properties that directly obtained from the mean flow equation.

The mean velocity gradient (∇U) is 2x2 tensor which will be flattened to become 4 input features for each component. The turbulent kinetic energy gradient (∇k) and mean pressure gradient (∇p) is a vector where each vector component will be the input features. Hence, we have 2 input features from turbulent kinetic energy gradient and 2 input features mean pressure gradient. In total, the FFNN model take 8 input features in the input layer. From the input features the model will predict 4 components of Reynolds stress tensor. The schematic of FFNN model proposed in this work could be seen in Figure 3.

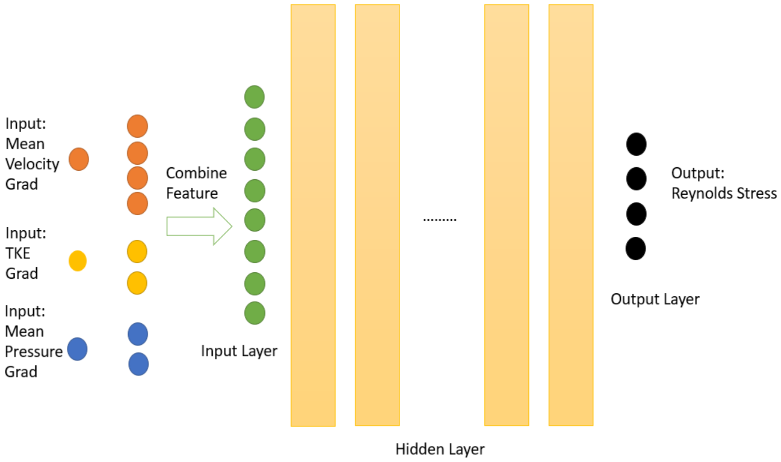

Figure 3 FFNN model architecture

The full RANS and DNS dataset are obtained from the Nature journal dataset by McConkey et al. [15]. The dataset consists of 5 cases which will be split into two datasets. 3 cases will be used as training data and 2 cases will be used as testing data. The training data will be used to train the neural network model and the testing dataset will be used to assess the performance of the model. The dataset split is given in Table 1.

Table 1 Dataset split

No	Data	Description
1	Training Data	$\alpha = 0.5, 0.8, 1.5$
2	Testing Data	$\alpha = 1.0, 1.2$

All input features and output features of the model will be standardized using standardization technique below,

$$x_{std} = \frac{x-\mu}{\sigma} \quad (5)$$

All the results of this work will be shown using the standardized value.

The hyperparameters used in developing the FFFN model are given in Table 2.

Table 2 FFNN model hyperparameter.

No	Hyperparameter	Description
1	Number of hidden layers	10
2	Number of neurons	20
3	Activation function	Tanh
4	Optimizer	Adam
5	Learning rate	0.001

4 Results and Analysis

From the model that has been developed, we obtain the neural network iteration process to minimize the loss function as shown in Figure 4,

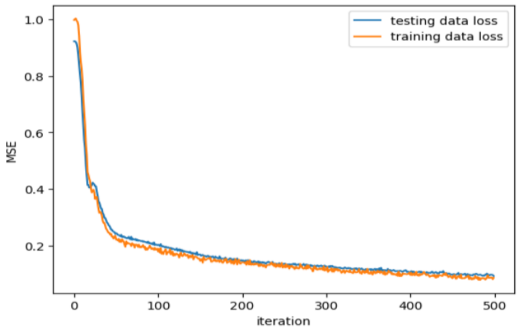

Figure 4 Neural network model training process

Figure 4 shows that the FFNN model already approaches the optimum solution in the training process, and it also gives accurate solution in the testing data. Hence, the model does not overfit the training data.

To assess the performance of each model, we use the root-mean-squared error (RMSE) between the Reynolds stress prediction of RANS or FFNN model and Reynolds stress from DNS. The comparison of RMSE between the RANS and FFNN model on the testing data is given in Table 3.

Table 3 RMSE of RANS and FFNN model

No	Model	Value
1	RANS	0.74
2	FFNN	0.32

From the Reynolds stress prediction, we could obtain the Reynolds stress field for the case of $\alpha = 1.0$ as shown in Figure 5,

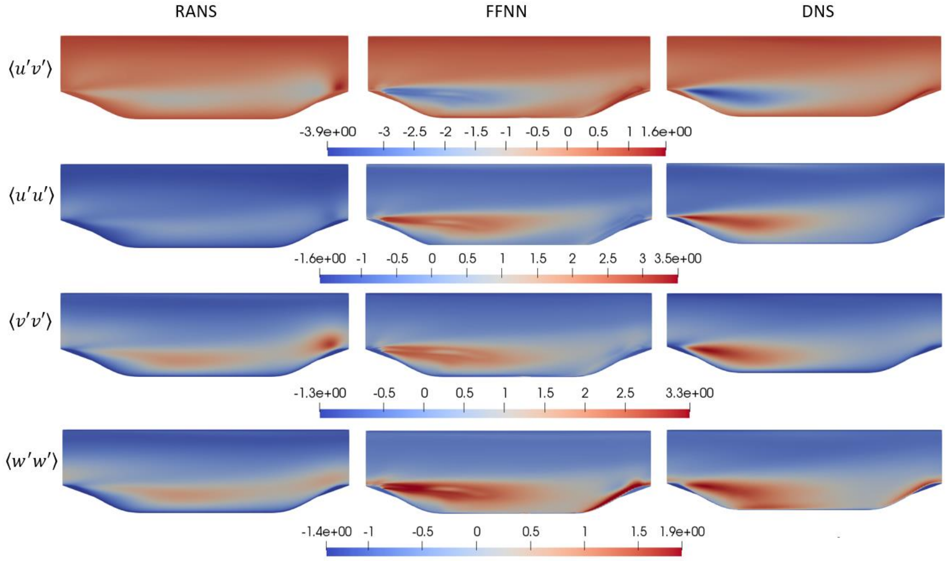

Figure 5 Comparison of Reynolds stress

From this result, the FFNN model could give better Reynolds stress prediction compared to the RANS model which gives 57% RMSE improvement from the RANS model. The main challenge to model a turbulent flow over periodic hills is on the appearance of flow separation in the flow-field. From Figure 1, the Reynolds stress field shows that the FFNN model could give quite accurate Reynolds stress prediction at the separation location where the RANS model is struggling to model this phenomenon.

5 Conclusion

In this work, a data-driven neural network model is developed for solving turbulence closure model problem. The neural network model is compared to direct numerical simulation (DNS) results as the benchmark for high-fidelity solution and also compared with the common model used in computational fluid dynamics (CFD). RANS $k - \omega$ model is member of linear Eddy-viscosity model (LEVM) from two-equation model class and used in this work as the common CFD model. From the results, the neural network model proposed in this work could give better turbulence closure prediction compared to the RANS $k - \omega$ model for flow over periodic hills which gives 57% RMSE improvement from the RANS model and it can model the Reynolds stress at the location of flow separation.

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