Proceedings of the 3rd ITB Graduate School Conference ISSN: 2963-718X
Enhancing Creativity in Research Through Developing Innovative Capabilities
December 21, 2022

On the Field Localization in a Thick Braneworld

Muhammad Taufiqur Rohman*, Triyanta & Agus Suroso

Theoretical High Energy Physics Division, Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung
*Email: mtrohman@students.itb.ac.id

Abstract. A braneworld theory is a higher dimension theory in which a 4D universe is assumed to be confined on a hypersurface embedded in the higher dimensional bulk. This theory is a good candidate that provides solution to the hierarchy problem in high-energy physics. In its development, the proof regarding complete field (such as massless and massive scalar, vector, and spinor) localization for several brane models has not been solved. In this article, we compared between thin and thick branes regarding the localization mechanism. We also analyzed the mechanism of field localization in two types of branes. The requirements for obtaining fields localization on thin branes are also examined.

Keywords: *braneworld; localization; weak field; Randall-Sundrum; thick brane.*

1 Introduction

The braneworld model, as one of the applications of the D-brane concept in string theory, provides a solution to the hierarchy problem in high-energy physics. The extra dimension is connected to the 4D universe by an exponential factor, called a warp factor. Any weak (TeV scale) fields or matter are assumed to be confined on a hypersurface, called a brane, embedded in a higher dimensional spacetime called a bulk. There are many models of brane that had been proposed. Some of the most popular are Randall-Sundrum (RS) [1]-[2], and its modified models [3].

Based on the characteristics of the warp factor, the brane models are classified into thin brane and thick brane. The thick brane model provides a more logical explanation regarding the minimal length scale in fundamental physics. The thick brane solutions are smooth. It can be applied in some higher-order derivative gravity theories. Meanwhile, the thin brane is described as a Dirac delta function in extra coordinates. Some of the thin brane models include Randall-Sundrum I and II, RS-like, and Modified Randall-Sundrum (MRS) by Jones, *et.al* in [3]. The idealistic thin brane model is still incomplete regarding field localization. The proposed MRS model provided better fields localization properties than RS model. Several studies in MRS have been conducted to examine the localization of scalar, vector, and spinor fields, each of them are massive and massless, by Wulandari, *et.al* in [5]-[8], and Rohman, *et.al* in [9].

In the case of thin branes, the localization mechanism follows Bajc *et.al* [4] and Jones *et.al* [3]. In this case, the separation of variables method is assumed and two conditions, namely the normalization and the mass term conditions, that enable ones to reduce the 5D into 4D action must be fulfilled.

In the case of thick brane, we consider a standard thick brane generated by scalar bulk, among them as in Liu, *et.al* [10]-[12]. In this case, the action is defined as 5D Einstein-scalar action with minimal coupling. By varying the action, the equations for the gravitational field and scalar field are obtained. From the field equations, the warp factor $A(y)$ and scalar field $\phi(y)$ maybe obtained after choosing and applying a mechanism such as a first-order formalism for the potential $V(\phi)$. In terms of localization, the warp factor plays an important role in field localization equation, called Schrödinger-like equation. The localization of the field $\phi(y)$ is defined as the following. If the energy of the field $\phi(y)$ is less than a critical energy the field is said to be localized. And vice versa, if its energy is greater than the critical energy it is said not localized. In the latter case the field propagates freely throughout the extra dimension. A paper by Dzhunushaliev *et.al* [14], as an example, discusses such a thing.

We assume the two scalar fields in thick and thin branes play two different roles. The scalar field that generates a thick brane is part of gravity, while the field in a thin brane is a matter or weak field that must be localized in the brane. So, the warp factor $A(y)$ obtained from the action of thick branes cannot be used in the normalization and mass equations in the case of the thin brane. This is because the general solution of the field cannot be obtained.

In Section 2, we will consider deriving the scalar and vector field localization equations in thin brane. In Section 3, we will discuss field localization in MRS-like, because we will determine the solution $A(y)$ of the scalar/vector field equations in extra dimension. Since we are looking for a localized field, the function $\chi(y)$ will be given specifically. In Section 4, we discuss field localization in thick branes. In addition, we also discuss the possibility of solution $A(y)$ in Section 3 when applied in this thick brane. In Section 5, discussion and conclusions.

2 Weak fields equations

Localization of several types of field is one of some important issues in braneworld. Here we consider a general metric of 5D braneworld system

$$ds^2 = p^2(x^5)\tilde{g}_{\mu\nu}dx^\mu dx^\nu - q^2(x^5)(dx^5)^2 \tag{1}$$

where $\tilde{g}$ is the 4D brane metric and x^5 is the extra dimension coordinate. The factors $p(x^5)$ and $q(x^5)$ represent warp factors for brane and extra dimension coordinates, respectively. Several models of 5D braneworld have been developed. In RS model [2], it is recognized that $p(y) = e^{A(y)}$ and $q = 1$. In MRS model [3], the coordinate transformation for extra dimension $dz = e^{-A(y)}dy$ was applied, so that the metric has $p(z) = q(z) = e^{A(z)}$ for z-coordinate system. Based on this metric transformation, MRS model is better than RS model on localization properties of scalar, vector, and spinor weak fields, whether they are massless or massive. For the spinor field, its localization in the MRS model is possible when the gamma matrices in a curved space, rather than in a flat space, is taken into account [6]. In this section, we will review the localization requirements that must be met by scalar and vector fields.

2.1 Scalar field

An action of 5D scalar matter field $\phi(x^M)$ with mass m_s is given by

$$S_0 = \frac{1}{2}\int d^5x\sqrt{g}\left(g^{MN}\partial_M\phi\partial_N\phi - m_s^2\phi^2\right) \tag{2}$$

where M, N indices represent 5D coordinate system, g is a determinant of the metric tensor g_{MN} that is defined in equation (1). Following the localization mechanism used by Bajc, *et.al* [3] and Jones, *et.al* [4], the 5D scalar field action can be reduced to the 4D scalar field action by field decomposition.
The decomposition for the scalar field is

$$\phi(x^\mu, x^5) = \sum_n \varphi_n(x^\mu)\chi_n(x^5). \tag{3}$$

leads the action (2) into

$$S_0^{(5D)} = \frac{1}{2}\int d^4x\sqrt{\tilde{g}}\tilde{g}^{\mu\nu}\partial_\mu\varphi_m\partial_\nu\varphi_n\int_{-\infty}^{\infty} dx^5p^2q\chi_m\chi_n$$
$$-\frac{1}{2}\int d^4x\sqrt{\tilde{g}}\varphi_m\varphi_n\int_{-\infty}^{\infty} dx^5\left[p^4q^{-1}(\partial_5\chi_m)(\partial_5\chi_n) + p^4qm_s^2\chi_m\chi_n\right].$$

This 5D action reduces to the 4D one, namely

$$S_0^{(4D)} = \frac{1}{2}\int d^4x\sqrt{\tilde{g}}\delta_{mn}\left(\tilde{g}^{\mu\nu}\partial_\mu\varphi_m\partial_\nu\varphi_n - m_m m_n\varphi_m\varphi_n\right). \tag{4}$$

The normalization equation

$$N_0 = \int_{-\infty}^{\infty} dx^5p^2q\chi_m(x^5)\chi_n(x^5) = \delta_{mn} \tag{5}$$

and 4D mass equation

$$m_0^2 = m_m m_n \delta_{mn} = \int_{-\infty}^{\infty} dx^5 \left[p^4 q^{-1}(\partial_5\chi_m)(\partial_5\chi_n) + p^4 q m_s^2 \chi_m \chi_n\right],$$
$$m_0^2 = \int_{-\infty}^{\infty} dx^5 \left[\frac{p^4}{q}(\partial_5\chi)^2 + p^4 q m_s^2 \chi^2\right]; \quad \text{for } m = n\ . \tag{6}$$

These two equations are called the properties or the conditions of localization. The equations (5) and (6) contain $\chi_n(x^5)$. This function should be in accordance with the scalar field equation, which is obtained by variation of the action S_0 with respect to the scalar field ϕ,

$$\frac{1}{\sqrt{g}}\left(\sqrt{g}g^{MN}\partial_N\phi\right) - m_s^2\phi = 0. \tag{7}$$

For a spacetime metric (1), the field equation can be derived as follows

$$\frac{1}{\sqrt{\tilde{g}}}\partial_\mu\left(\sqrt{\tilde{g}}\tilde{g}^{\mu\nu}\partial_\nu\phi\right) - \frac{p^2}{q^2}\left[\partial_5^2\phi + \left(4\frac{\partial_5 p}{p} - \frac{\partial_5 q}{q}\right)\partial_5\phi\right] - p^2 m_s^2\phi = 0. \tag{8}$$

Imposing the decomposition (4) and considering φ_n as 4D scalar field satisfying the massive 4D Klein-Gordon, $\frac{1}{\sqrt{\tilde{g}}}\left(\sqrt{\tilde{g}}\tilde{g}^{\mu\nu}\partial_\nu\varphi_n\right) = m_n^2\varphi_n$, then the field equation above reduces to equation for $\chi_n(x^5)$

$$\partial_5^2\chi_n + \left(4\frac{\partial_5 p}{p} - \frac{\partial_5 q}{q}\right)\partial_5\chi_n + q^2 m_s^2\chi_n - \frac{q^2}{p^2}m_n^2\chi_n = 0. \tag{9}$$

2.2 Vector field

Consider a 5D vector field

$$A_M(x^M) = \left(A_\mu(x^M), A_5\right) = \left(\sum_n a_\mu^{(n)}(x^\mu)\alpha_{(n)}(x^5), A_5\right) \tag{10}$$

where A_5 is chosen as a constant or zero [13]. The action of 5D vector field is given by

$$S_1 = -\frac{1}{4}\int d^5x\sqrt{-g}g^{MN}g^{RS}F_{MR}F_{NS}$$
$$= -\frac{1}{4}\int dx^5\sqrt{g}g^{\mu\nu}g^{\rho\sigma}\alpha_{(n)}^2\int d^4x f_{\mu\rho}^{(n)}f_{\nu\sigma}^{(n)} - \frac{1}{2}\int dx^5\sqrt{g}g^{55}g^{\rho\sigma}(\partial_5\alpha_{(n)})^2\int d^4x\, a_\rho^{(n)}a_\sigma^{(n)}.$$

with the field strength tensors $F_{MN} = \partial_M A_N - \partial_N A_M$, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \alpha_{(n)}(\partial_\mu a_\nu^{(n)} - \partial_\nu a_\mu^{(n)}) = \alpha f_{\mu\nu}^{(n)}$ is equivalent to

$$S_1^{(5D)} = -\frac{1}{4}\sum_n \left[\int dx^5 q\alpha_{(n)}^2 \int d^4x\sqrt{\tilde{g}}\tilde{g}^{\mu\nu}\tilde{g}^{\rho\sigma} f_{\mu\rho}^{(n)} f_{\nu\sigma}^{(n)} + 2\int dx^5 \frac{p^2}{q}(\partial_5\alpha_{(n)})^2 \int d^4x\sqrt{\tilde{g}}\tilde{g}^{\rho\sigma} a_\rho^{(n)} a_\sigma^{(n)} \right]$$

when the metric (1) is considered. This 5D action will reduce to the 4D one

$$S_1^{(4D)} = -\frac{1}{4}\sum_n \int d^4x\sqrt{\tilde{g}}\tilde{g}^{\mu\nu}\tilde{g}^{\rho\sigma} f_{\mu\rho}^{(n)} f_{\nu\sigma}^{(n)} - \frac{1}{2}\sum_n m_{1(n)}^2 \int d^4x\sqrt{\tilde{g}}\tilde{g}^{\rho\sigma} a_\rho^{(n)} a_\sigma^{(n)} \quad (11)$$

if the following normalization and mass equations

$$N_1 = \sum_n \int dx^5 q\alpha_{(n)}^2; \quad m_1^2 = \sum_n \int dx^5 \frac{p^2}{q}\left(\partial_5\alpha_{(n)}\right)^2 \quad (12)$$

are satisfied. It can be proven that the action S_1 and the metric (1) give rise to the field equations

$$\frac{1}{\sqrt{\tilde{g}}}\partial_\mu\left(\sqrt{\tilde{g}}\tilde{g}^{\mu\nu}\tilde{g}^{\rho\sigma} f_{\nu\sigma}^{(n)}\right) + \frac{a_{(n)}^\rho}{\alpha_{(n)}}\frac{1}{q}\partial_5\left(\frac{p^2}{q}\partial_5\alpha_{(n)}\right) = 0. \quad (13)$$

Consider two types of vector fields, a massive field and massless field. A 4D massive vector field $f_{\mu\nu}$ satisfying the Proca equation

$$\frac{1}{\sqrt{\tilde{g}}}\partial_\mu\left(\sqrt{\tilde{g}}\tilde{g}^{\mu\nu}\tilde{g}^{\rho\sigma} f_{\nu\sigma}^{(n)}\right) + m_{(n)}^2 a_{(n)}^\rho = 0 \quad (14)$$

leads to the equation for α_n

$$\frac{1}{q}\partial_5\left(\frac{p^2}{q}\partial_5\alpha_{(n)}\right) - m_{(n)}^2\alpha_{(n)} = 0. \quad (15)$$

For a 4D massless vector field, the Proca equation reduces to the Maxwell equation and the equation for α_n simplifies into

$$\frac{1}{q}\partial_5\left(\frac{p^2}{q}\partial_5\alpha_{(n)}\right) = 0. \quad (16)$$

3 Fields confinement on MRS-like

The MRS model, with z-coordinate system, has better properties of localization than the y-coordinate system in the RS model. In addition, the z-coordinate system has a proper distance for extra dimensions whose value is finite [3]. In the case of the MRS brane model with the warp factor $p(z) = q(z) = e^{A(z)}$, the normalization, mass, and field equations for scalar and vector fields are given as follows.

1. Scalar field

$$N_0 = \int_{-\infty}^{\infty} dz e^{3A(z)} \chi^2(z) = 1$$
$$m_0^2 = \int_{-\infty}^{\infty} dz \left[e^{3A(z)} (\partial_z \chi(z))^2 + e^{5A(z)} m_s^2 \chi^2(z) \right], \tag{18}$$

$$\partial_z^2 \chi(z) + 3\partial_z A(z) \partial_z \chi(z) - m^2 \chi(z) = 0; \quad (m_s = 0). \tag{19}$$

2. Vector field

$$N_1 = \int_{-\infty}^{\infty} dz e^{A(z)} \alpha^2(z); \quad m_1^2 = \int_{-\infty}^{\infty} dz e^{A(z)} \left(\partial_z \alpha(z)\right)^2 \tag{20}$$

$$\partial_z^2 \alpha(z) + \partial_z A(z) \partial_z \alpha(z) - m^2 \alpha(z) = 0. \tag{21}$$

We have a freedom to choose either $A(z)$ and $\chi(z)$ in (19) also $A(z)$ and $\alpha(z)$ in (21). In the following we will consider some simple choices.

The MRS model as a thin brane explicitly has the warp factor $A = -k|z|$, where k is a constant. This form of the warp factor is simple, so that it is easy to obtain general solution of $\chi(z)$ for all range of z. This model is better in terms of fields localization as compared to the RS model. However, it is worth to choose some forms of $A(z)$ explicitly that hopefully lead to a complete localization of the field. The given $A(z)$ must be used in the field equation (19) to find a solution $\chi(z)$. Then, the given $A(z)$ and the obtained $\chi(z)$ must satisfy the equation (18). From those equations, it is known that to obtain the finite value of the integral, the integrand must be convergent for the whole range of z.

Consider two functions F and G, for spin-0 and spin-1 fields

$$F_0(z) = e^{3A(z)} \chi^2(z), \quad G_0(z) = e^{3A(z)} (\partial_z \chi(z))^2$$
$$F_1(z) = e^{A(z)} \alpha^2(z), \quad G_1(z) = e^{A(z)} (\partial_z \alpha(z))^2 . \tag{22}$$

These functions are the integrands of the integrals (18) and (20). Choosing $A(z)$ must ensure that the general solutions of $\chi(z)$ and $\alpha(z)$ can be obtained. The functions F_s and G_s (for $s = 0,1$) must satisfy the following conditions

$$\int_{-\infty}^{\infty} F_s(z) dz < \infty, \quad \int_{-\infty}^{\infty} G_s(z) dz < \infty. \tag{23}$$

The integrals converge if the function F_s or G_s converges.

Choosing the function $\chi(z)$ is probably better than choosing the function $A(z)$. This is because we can choose the function $\chi(z)$ so that its values dominate around the brane, thus in accordance with the aim of finding a localized field. In addition, to find solution $A(z)$, it is enough to solve the first-order differential equation (19), provided that at least the field function must be up to second-order differentiable.

A constant function of $\chi(z)$

Consider a constant $\chi(z)$. From the field equation (19), we get $m = 0$. It means that a constant field only valid for a massless scalar field. From equation (18), it also obtained that $m_s = 0$. Then, there is only an equation to determine the warp factor, that is

$$N_0 = \int_{-\infty}^{\infty} dz e^{3A(z)} = \chi^{-2} = \text{const}. \quad (24)$$

In this case, the warp factor $A(x^5)$ must satisfy equation (24).

A Gaussian function of $\chi(z)$

Consider a function $\chi(z) = e^{-az^2}$, where a is a constant. From the equation (19), a solution for the warp factor is

$$A(z) = \frac{1}{3}az^2 - \left(\frac{1}{3} + \frac{m^2}{6a}\right)\ln(z) + c\,. \quad (25)$$

For simplicity, we take $c = 0$ and $a = m = 1$. The normalization equation in (18) gives a convergent value. Meanwhile, from the mass equation in (18) gives a divergent value. So, the corresponding scalar field are not localized on the brane.

4 Thick brane generated by a scalar field

Consider a thick brane generated by a scalar bulk. The system is based on 5D Einstein-scalar gravity, where the action is given by

$$S_g = -\frac{1}{4}\int d^5x\sqrt{g}R + S_{\text{scalar}} = \int d^5x\sqrt{g}\left(-\frac{R}{4} + \frac{1}{2}g^{MN}\partial_M\phi\partial_N\phi - V(\phi)\right) \quad (26)$$

where R is Ricci scalar and $V(\phi)$ is a potential for scalar field $\phi(x^5)$. By varying the action (26) with respect to the metric g^{MN}, the Einstein field equation is obtained,

$$G_{MN} \equiv R_{MN} - \frac{1}{2}g_{MN}R = T_{MN} \quad (27)$$

where the energy-momentum tensor is

$$T_{MN} = \frac{-2}{\sqrt{g}} \frac{\delta S}{\delta g^{MN}}. \tag{28}$$

Consider a 5D spacetime with metric (1) and Minkowskian brane for simplicity. The components of Einstein tensor are

$$\begin{aligned} G_{\mu\nu} &= \eta_{\mu\nu} 3 \frac{p^2}{q^2} \left(\frac{p'q'}{pq} - \frac{p''}{p} - \frac{p'^2}{p^2} \right) \\ G_{55} &= 6 \frac{p'^2}{p^2} \end{aligned} \tag{29}$$

while non-zero components of the energy-momentum tensor are

$$\begin{aligned} T_{\mu\nu} &= \eta_{\mu\nu} \frac{p^2}{q^2} \left(\frac{1}{2} \phi'^2 + q^2 V \right) \\ T_{55} &= \frac{1}{2} \phi'^2 - q^2 V. \end{aligned} \tag{30}$$

Then, the components of Einstein equation are

$$\begin{aligned} (\mu, \nu): &\qquad 3 \left(\frac{p'q'}{pq} - \frac{p''}{p} - \frac{p'^2}{p^2} \right) = \frac{1}{2} \phi'^2 + q^2 V \\ (5,5): &\qquad 6 \frac{p'^2}{p^2} = \frac{1}{2} \phi'^2 - q^2 V. \end{aligned} \tag{31}$$

***y*-coordinate system.** In y-coordinate system, $p(y) = e^{A(y)}$ and $q = 1$, the Einstein equations become

$$\begin{aligned} (\mu, \nu): &\qquad -3 \left(A'' + 2A'^2 \right) = \frac{1}{2} \phi'^2 + V(\phi) \\ (y, y): &\qquad 6A'^2 = \frac{1}{2} \phi'^2 - V(\phi). \end{aligned} \tag{32}$$

or they can be rewritten as

$$A'' = -\frac{1}{3} \phi'^2, \quad A'^2 = \frac{\phi'^2}{12} - \frac{V}{6}. \tag{33}$$

If the two equations are combined by eliminating the kinetic term, the potential is obtained

$$V = -\frac{3}{2} \left(A'' + 4A'^2 \right). \tag{34}$$

An appropriate warp factor can be inserted in this potential. In this coordinate system, we have to make the warp factor transformation from z to y-coordinate, $A(z) \rightarrow A(y)$, using

$$y = \int e^{A(z)} dz. \tag{35}$$

To get z as a function of y it is necessary to find the inverse of the function $y(z)$ in (35). Then, $z(y)$ is substituted into $A(z)$. If $A(z)$ considered gives the complete localization substituting into (34) will give an explicit form of potential that describes the braneworld gravitational system that localizes the weak fields.

z-coordinate system: In this coordinate, $p(z) = q(z) = e^{A(z)}$. The Einstein equations are

$$\begin{aligned} (\mu,\nu): \quad & -3\left(A'' + A'^2\right) = \frac{1}{2}\phi'^2 + e^{2A}V \\ (z,z): \quad & 6A'^2 = \frac{1}{2}\phi'^2 - e^{2A}V \end{aligned} \tag{36}$$

where

$$V = -\frac{3}{2}e^{-2A}\left(A'' + 3A'^2\right). \tag{37}$$

Without changing the coordinate system, solution $A(z)$ can be directly used in this equation.

4.1 Field localization mechanism in a thick brane

The warp factor solution is an important variable that determines the localization conditions of weak fields in the brane. In this section, the localization mechanism for a thick brane is given in case of the scalar field.

Consider a 5D real scalar field, $\phi(x^\mu, r)$,

$$S_0 = -\frac{1}{2}\int d^5x\sqrt{-g}\left(g^{MN}\partial_M\phi\partial_N\phi + m_{(5)}^2\phi^2\right) \tag{38}$$

where $m_{(5)}$ is a bulk scalar mass, g is a determinant of the metric tensor g_{MN}. By varying the action with respect to the scalar field, an equation of motion for a scalar field is obtained

$$\frac{1}{\sqrt{-g}}\partial_M\left(\sqrt{-g}g^{MN}\partial_N\phi\right) + m_{(5)}^2\phi = 0. \tag{39}$$

For a spacetime with metric

$$ds^2 = e^{2A}\left[\tilde{g}_{\mu\nu}dx^\mu dx^\nu - dz^2\right]. \tag{40}$$

The equation of motion for scalar field can be rewritten as

$$\frac{1}{\sqrt{-\tilde{g}}}\partial_\mu\left(\sqrt{-\tilde{g}}\tilde{g}^{\mu\nu}\partial_\nu\phi\right) + \phi'' + 3A'\phi' - m_{(5)}^2 e^{2A}\phi = 0. \tag{41}$$

Applying the Kaluza-Klein (KK) decomposition, $\phi(x^\mu, z) = \sum_n \varphi_n(x^\mu)\chi_n(z)$, and imposing that the 4D scalar field obeys 4D Klein-Gordon equation, then the field equation for scalar bulk ($m_{(5)} = 0$) is obtained

$$\chi_n'' + 3A'\chi_n' + m_n^2\chi_n = 0, \tag{42}$$

where m_n is KK mass spectrum. As described in the case of thin brane, the reduction of 5D action to 4D action requires normalization and mass conditions. In the articles discussing thick brane, these conditions were not tested further. In the case of thick branes, the field equations are transformed into Schrödinger-like equation, by defining a new scalar field

$$\tilde{\chi}_n(z) = e^{\frac{3}{2}A}\chi_n(z). \tag{43}$$

The equation for 5D scalar field is

$$\left[-\partial_z^2 + V_0(z)\right]\tilde{\chi}_n = m_n^2\tilde{\chi}_n; \qquad V_0(z) = \frac{3}{2}A'' + \frac{9}{4}A'^2, \tag{44}$$

where $V_0(z)$ is the effective potential of the scalar field. It depends on the warp factor only. The localization condition can be analyzed from the mass spectrum of the scalar field. A state related to the mass m_i must have an energy lower than the critical energy, to satisfy the field localization on the brane. A massless mode as an analogy to the ground state, and some massive modes as an analogy to the excited states.

5 Discussion and conclusions

By definition, a braneworld traps weak fields (TeV scale) in a brane as a hypersurface. As stated in previous studies, in RS brane model not all fields are well localized. For this reason, we tried to conduct a review regarding the localization of the fields. As is well known, there are two types of branes based on the characteristics of the warp factor, those are thick and thin branes. The two types of branes also describe a localization mechanism that is technically different, but fundamentally has the same mechanism. First, thin branes try to analyze localization which focuses on reducing 5D action to 4D action. Then, the normalization and mass equations are obtained. Weak scale fields must obey these equations to be said to be locally in the brane. Second, thick branes analyze the localization of the field equations in the extra coordinates, which are called Schrödinger-like equations. The bound states (ground state and some excited states) in the system are analogous to a localized field with a certain mass mode.

Solution $A(x^5)$ obtained from the gravitational field equation for thick branes cannot be used for the normalization and mass equations for thin brane. If the solution $A(x^5)$ is applied in the field equation, then the general solution for the field cannot be obtained. Due to the impossibility of obtaining the general solution, we emphasize the review of the normalization and mass equations directly to obtain the solution conditions $A(x^5)$ or $\chi(x^5)$. As was done in Section 3, it would be better if explicit function of either $A(x^5)$ or $\chi(x^5)$ is given. Between the two, choosing the field function would be better, because we can choose a function that describes the presence of a field at a certain point or area. Physically, the Dirac delta function would be a good candidate, because it represents the field function in a thin brane. However, it becomes a problem when the Dirac delta function is applied to the field equations to obtain $A(x^5)$.

From the possibility that if we obtain $A(x^5)$ that meets the requirements in Section 3, then this $A(x^5)$ can be used to define braneworld metric. We have considered the bulk scalar brane system in Section 4 as the gravitational framework.

Acknowledgement

This research was supported by PPMI-ITB.

References

[1] Randall, L. & Sundrum, R., *Large mass hierarchy from a small extra dimension*, Physical Review Letters **83** (10) pp. 3370-3373, 1999.

[2] Randall, L., & Sundrum, R., *An Alternative to Compactification*, Physical Review Letters, **83**(23), pp. 4690-4693, 1999.

[3] Jones, P., Muñoz, G., Singleton, D., & Triyanta, *Field localization and the Nambu–Jona-Lasinio mass generation mechanism in an alternative five-dimensional brane model*, Physical Review D, **88**, 025048, 2013.

[4] Bajc, B. & Gabadadze, G., *Localization of matter and cosmological constant on a brane in Anti-de Sitter space*, Physics Letters B, 474 pp. 282-291, 2000.

[5] Wulandari, D., Triyanta, Kosasih, J. S. & Singleton, D., *Localization of a system of vector and scalar fields in a modified Randall-Sundrum model*, AIP Conference Proceedings, 040003, 2015.

[6] Wulandari, D., Triyanta, Kosasih, J. S., Singleton, D. & P. Jones, *Localization of interacting fields in five-dimensional braneworld models*, International Journal of Modern Physics A, 32, 2017.

[7] Wulandari, D. Triyanta, Kosasih, J. S. & Singleton, D., *Localization of matter fields that interact with gravity in a non-minimal derivative coupling in the Randall-Sundrum model*, Journal of Physics: Conference Series **1120** (11) 012040, 2018.

[8] Wulandari, D., Triyanta, Kosasih, J. S. & Singleton, D., *The Field Localization of Yukawa Interaction in a Modified Randall-Sundrum Model*, Journal of Physics: Conference Series, 2019.

[9] Rohman, M. T. & Triyanta, *Searching for a modification of a conformally flat metric in the five-dimensional braneworld model that localizes scalar and vector fields*, Journal of Physics: Conference Series, **1949** (6) 012004, 2021.

[10] Liu, Y., *Introduction to extra dimensions and thick braneworlds*, Memorial volume for Yi-shi Duan, pp. 211-275, 2018.

[11] Liu, Y., Guo, H., Fu, C., & Li, H., *Localization of gravity and bulk matters on a thick anti-de Sitter brane*, Physical Review D, **84**(4), 044033, 2011.

[12] Liu, Y., Zhang, L., Wei, S., & Duan, Y., *Localization and mass spectrum of matters on Weyl thick branes*, Journal of High Energy Physics, 2008 (08), 041, 2008.

[13] Guo, H., Herrera-Aguilar, A., Liu, Y., Malagón-Morejón, D., & Mora-Luna, R., *Localization of bulk matter fields, the hierarchy problem and corrections to Coulomb's law on a pure de Sitter thick braneworld*, Physical Review D, 87 (9), pp.1-21, 2013.

[14] Dzhunushaliev, V., Folomeev, V. & Minamitsuji, M., *Thick brane solutions*, Reports on Progress in Physics, **73** (2010) 066901.